Dec 9

Note Title 12/9/2009

$$S = \{(x, g(x)) : x \in [0,1]\}$$

g - cont

Show S is compact.

1) Closed & bounded

2) Every segn. has a conv. subsegu. with limit in set.

3) ~~Every open~~ cover has ~~a finite~~ subcover.

Let's show S is closed & bounded.

bounded.

1) $(x,y) \in S$,

$|x| \leq 1$

$y = g(x)$

g - cont fct.

on closed bdd ~~int~~

so g has a

max + min & is

bounded.

i.e. $\exists B$, $|g(x)| \leq B$.

so $\|(x, y)\| \leq B + 1$

Closed.

S is closed if

every (x_i, y_i) from

S that converges

converges to a pt

of S.

Suppose $(x_i, y_i) \in S$

& conv. to (x, y)

i) $x_i \in [0, 1]$

$x_i \to x$.

& $x \in [0, 1]$

as it is closed.

$y_i = g(x_i)$

so $y_i \to g(x)$

as g is cont.

but $y_i \to y = g(x)$

& $(x, y) = (x, g(x)) \in S$.

4) If S is the continuous image of a set I known is cpct then S is cpct.

$[0,1] \subseteq \mathbb{R}$ is cpct as closed & bded.

$G(x) = (x, g(x))$ – cont. as coord. are.

so $G([0,1])$ is cpct.

$G([0,1]) = \{(x, g(x)) : x \in [0,1]\}$.

2)

$$G(x,y) = F(2F(x,y), F(x,y))$$

$$F(0,0) = 0$$

$$G(0,0) = \underline{F(0,0) = 0.}$$

$$G(x,y) = F(u(x,y), v(x,y))$$

$$u(x,y) = 2F(x,y)$$

$$v(x,y) = \underline{F(x,y)}$$

$$\partial_x G = \partial_u (F(u,v))\, \partial_x u$$

$$+$$

$$\underline{\partial_v (F(u,v))\, \partial_x v}$$

$$= \partial_u (F(0,0))\, 2\, \partial_x F(0,0)$$

$$+ \partial_v [F(0,0)]\, \partial_x F(0,0)$$

$$= 2(\partial_x \overline{F(0,0)})^2$$

$$+ \partial_y (F(0,0))\, \partial_x F(0,0)$$

$$\partial_x F(0,0) = 1$$

$$\text{So} = 2 + \partial_y F(0,0) = 0$$

$$\partial_y F(0,0) = -2$$

3) $f : (a,b) \to \mathbb{R}$

$x_0 \in (a,b)$.

Taylor series at x_0

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!} (x-x_0)^n.$$

$$f^{(n)}(x_0) \leq M^n.$$

$$\left| \frac{f^{(n)}(x_0)}{n!} (x-x_0)^n \right|$$

$$\leq \left| \frac{M^n (x-x_0)^n}{n!} \right|$$

$$= \frac{a^n}{n!}, \quad a = |M(x-x_0)|$$

$$\& \quad \sum_{n=1}^{\infty} \frac{a^n}{n!} = e^a$$

For all $a \in \mathbb{R}$.

so as these terms dominate the terms of the given series it also converges.

$e^{-\frac{1}{x^2}}$ 0 at 0

$|x|^{\frac{1}{|x|}}$ 0 at 0

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4) $\begin{pmatrix} f_1 \\ f_2 \end{pmatrix} = \begin{pmatrix} e^{x+y+z} \\ x^2+y^2+z^2 \end{pmatrix}$

(a, b, c)

$F(x, y, z) - \underline{F(a, b, c)} = 0$

$$\begin{bmatrix} e^{y+z+x} & e^{x+z+y} & e^{x+y+z} \\ 2x & 2y & 2z \end{bmatrix} \quad (a, b, c)$$

$$\begin{bmatrix} e^{a+b+c} & e^{a+b+c} & e^{a+b+c} \\ 2a & 2b & 2c \end{bmatrix}$$

$y \quad z$

Need
$$\begin{vmatrix} e^{a+b+c} & e^{a+b+c} \\ 2b & 2c \end{vmatrix} \neq 0$$

$$2(b-c)e^{a+b+c} \neq 0$$

$$\Leftrightarrow \underline{\underline{b \neq c}}$$