Note Title 8/24/2009

Aug 24

read carefully pages 1-3

Euclidean Spaces.

$\mathbb{R}^n - \mathbb{E}^n$

$\{(x_1, \dots x_n) : x_i \in \mathbb{R}\}$

\- Geometry -

2 notions -

length

angle

$$\vec{x} = (x_1, x_2, \ldots, x_n)$$

$$\|\vec{x}\| = (x_1^2 + \cdots + x_n^2)^{1/2}$$

angles?

dot product

inner product

$$\vec{u} = (u_1 \cdots u_n)$$

$$\vec{v} = (v_1 \cdots v_n)$$

$$\vec{u} \cdot \vec{v} = \langle u, v \rangle$$

$$= \sum_{i=1}^{n} u_i v_i$$

$$\|\vec{u}\|^2 = \vec{u} \cdot \vec{u}$$

Thm (Cauchy-Schwartz inequality)

$$|\vec{u}\cdot\vec{v}| \le \|\vec{u}\|\|\vec{v}\| \leftarrow$$

Pf. $f(t) = \|\vec{u} + t\vec{v}\|^2$

$(\vec{u} + t\vec{v}) \cdot (\vec{u} + t\vec{v})$

$= \underline{\vec{u}\cdot\vec{u} + 2t\,\vec{u}\cdot\vec{v} + t^2\,\vec{v}\cdot\vec{v}}$

1) if $\vec{v} = \vec{0}$ <u>true</u>

2) if $\vec{v} \neq 0$.

f is quadratic.

opening up.

So f has a unique min.

$$f'(t) = 2\vec{u}\cdot\vec{v} + 2t_0\vec{v}\cdot\vec{v} = 0$$

$$t_0 = -\frac{\vec{u}\cdot\vec{v}}{\vec{v}\cdot\vec{v}}$$

$$f(t) = \|\vec{u} + t\vec{v}\|^2 \geq 0$$

$f(t_0) \geq 0$ or

$$\vec{u}\vec{u} - 2\frac{\vec{u}\cdot\vec{v}}{\vec{v}\cdot\vec{v}}\vec{u}\cdot\vec{v} + \frac{(\vec{u}\cdot\vec{v})^2}{(\vec{v}\cdot\vec{v})^2}\vec{v}\cdot\vec{v}$$

$$\geq 0$$

$$\|\vec{u}\|^2 - \frac{(\vec{u}\cdot\vec{v})^2}{\|\vec{v}\|^2} \geq 0$$

$$(\vec{u}\cdot\vec{v})^2 \leq \|\vec{u}\|^2\|\vec{v}\|^2$$

$$|\vec{u}\cdot\vec{v}| \leq \|\vec{u}\|\,\|\vec{v}\| \quad \blacksquare$$

Corollary:

$$|\vec{u}\cdot\vec{v}| = \|\vec{u}\|\|\vec{v}\|$$

iff $\vec{u} = c\vec{v}$ for some $c \in \mathbb{R}$.

pf. $\vec{u} = c\vec{v}$

$$|\vec{u}\cdot\vec{v}| = |c\vec{v}\cdot\vec{v}|$$

$$= |c|\,\|\vec{v}\|^2$$

$$= |c|\,\|\vec{v}\|\,\|\vec{v}\|$$

$$= \underline{\|\vec{u}\|\,\|\vec{v}\|}$$

Suppose

$$\|u\cdot\vec{v}\| = \|\vec{u}\|\,\|\vec{v}\|$$

$$f(t) = \|u\|^2 + 2\vec{u}\cdot\vec{v}\,t + t^2\|v\|^2$$

$$= \|\vec{u}\|^2 + 2\|\vec{u}\|\|\vec{v}\|t + t^2\|\vec{v}\|^2$$

$$= \underline{(\|\vec{a}\| + t\|\vec{v}\|)^2}$$

$$f(t_0) = \|\vec{u}\|^2 - \frac{(\vec{u}\cdot\vec{v})^2}{\|\vec{v}\|^2}$$

$$= \|\vec{u}\|^2 - \frac{\|\vec{u}\|^2 \cancel{\|\vec{v}\|^2}}{\cancel{\|\vec{v}\|^2}} = 0$$

$$f(t_0) = \|\vec{u} + t_0\vec{v}\|^2 .$$

$$\Rightarrow \vec{u} + t_0\vec{v} = 0$$

& so $\underline{\vec{u} = -t_0\vec{v} = c\vec{v}}$.

Δ - inequality

$$\|\vec{a}+\vec{b}\| \leq \|\vec{a}\| + \|\vec{b}\|$$

$$(\vec{a}+\vec{b})\cdot(\vec{a}+\vec{b}) \qquad \vec{a}\cdot\vec{a} + 2\|\vec{a}\|\|\vec{b}\| + \vec{b}\cdot\vec{b}$$

$$\vec{a}\cdot\vec{a} + 2\,\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{b} \leq$$