Homework

M472 Fall 2014

Homework grading supreme laws:

1. *Thou shalt do and write up ALL homework problems.*
2. *Thou shalt select two problems, which shall be denoted "best" and "worst".*
3. *Whenceforth, the problem deemed "best" shall be graded, and the one deemed "worst" shall be given feedback.*
4. *Thou shalt not let your dog chew on the homework, or let the pristine aesthetic quality of your work be otherwise compromised!*

Important note: in this homework I use several times the words "smallest", "largest"... anytime I do not specify which ordering relation I refer to I am implicitely assuming I am talking about inclusion!

Exercise 1. *Recall the definition of the taxicab metric on the plane:*

$$d_{max}((x_1, y_1), (x_2, y_2)) = |x_2 - x_1| + |y_2 - y_1|$$

Considering the plane a metric space with this metric, draw an open ball of radius 1 *centered at* $(0, 0)$.

Exercise 2. *Prove that a subset of the plane is open in the topology induced by the taxi cab metric if and only if it is open in the topology induced by the euclidean metric.*

Exercise 3. *If* X *is a set with exactly three points, describe all possible topologies you can put on* X.

Exercise 4. *Let* X *be a topological space, let* $A \subseteq X$.

1. *Prove that the set of interior points of* A *is the largest open set contained in* A. *We call such set the* ***interior*** *of* A.
2. *Prove that the union of* A *with set of limit points of* A *is the smallest closed set containing* A. *We call such set the* ***closure*** *of* A.
3. *Prove that the complement of the interior of* A *equals the closure of the complement of* A.

Exercise 5. *Let* X *have the stupid topology, and* $x \in X$. *For what subsets of* X *is* x*:*

- *an interior point?*
- *a limit point?*

Exercise 6. *Consider* $X = Y = \mathbb{R}$ *and the function* $f : X \to Y$ *given by* $y = f(x) := [x]$ *(*y *is the integer part of* x*, e.g.* $f(2.4) = 2$*) Recall that* $\mathbb{R}$ *can be given a topology* τ_1 *by declaring open sets to be open half lines of the form* $(-\infty, a)$*, or it can be given a topology* τ_2 *by declaring open sets to be open half lines of the form* (a, ∞) *(and remember that in both cases you have to toss in the empty set and all of* $\mathbb{R}$ *as well). Consider all for possibile combinations of assigning a topology to* X *and to* Y *from* τ_1 *and* τ_2*, and tell me when* f *is continuous and when it is not.*