Homework

M472 Fall 2014

Exercise 1. *Is it true or false that if two topological spaces X and Y are such that there exists a bijection between X and Y and there exists a bijection between τ_X and τ_Y, then they are homeomorphic? If true, prove it. If false, provide a counter-example.*

Exercise 2. *Let $X = [0, 1]$, with euclidean topology, and let Y be the graph of the function $y = |x|$ restricted to the domain $-1 \leq x \leq 1$, with topology induced by the euclidean topology in $\mathbb{R}^2$. Prove, with great care and detail, that X and Y are homeomorphic.*

Exercise 3. *After exercise 2, now you can be a little bit more liberal in claiming homeomorphisms about "stick figure"-like objects. Subdivide into homeomorphism classes the letters of the alphabet. You don't need to motivate why things are homeomorphic. And we don't yet have refined enough tools to formally prove when things are not homeomorphic. But try nonetheless to find some "reasonable" motivations/guesses to argue why your equivalence classes are distinct. This is an exercise to try and come up with reasonable guesses for topological invariants, thus developing some intuition on how these things work.*

NOTE: since your answer obviously depends on how you write your letters, make sure that you write down the letters very carefully - and consistently!

Exercise 4. *Let X and Y be topological spaces and let $X \times Y$ be the product space (i.e. the cartesian product endowed with the product topology). Prove that for any $x \in X$ the subspace $\{x\} \times Y$ (endowed with the subspace topology) is homeomorphic to Y.*

Exercise 5. *Let $X = \mathbb{R} \setminus \{0\}$ (with euclidean topology), and $Y = \{\pm 1\}$ a SET with two elements. Consider the following functions f and $g : X \to Y$.*

$$f(x) = \begin{cases} 1 & \text{if } x < 0 \\ -1 & \text{if } x > 0 \end{cases}$$

$$g(x) = \begin{cases} 1 & \text{if } x \in \mathbb{Q} \\ -1 & \text{if } x \notin \mathbb{Q} \end{cases}$$

What is the finest topology on Y that makes f is continuous? What is the finest topology on Y that makes g is continuous?