Homework #9

M472 Fall 2012

Problem 1. *Let L be an m-dimensional linear subspace of $\mathbb{R}^n$. Show that $\mathbb{R}^n \setminus L$ is homotopy equivalent to S^{n-m-1} (an $n-m-1$ dimensional sphere).*

Hint: *first prove the case when L is just a single point (aka a 0-dimensional linear subspace). Then study the case of a line in three dimensional space and try to generalize that approach.*

Problem 2. *Choose your favorite of the three group axioms (associativity of the product, existence of an identity element, existence of an inverse) and prove that the fundamental group satisfies such axiom by writing down the relevant homotopy explicitly (yes, for once you do have to write down a formula $H(s,t) = \ldots$).*

Problem 3. *Consider the loop $\gamma : [0,1] \to S^1$ defined by $\gamma(s) = \theta = 2\pi s$ as an element of the fundamental group of the circle. Write down an expression for $\gamma \star \gamma$. How about $\gamma^{\star n}$ (where here I mean γ composed with itself n times)?*

Problem 4. *Let $X = Y = S^1$ and let $f : X \to Y$ be the function $f(\theta) = 3\theta$. Let $\gamma : [0,1] \to X$ be as before. What is $\pi_1(f)(\gamma)$?*

Problem 5. *Take as a fact that the fundamental group of S^1 is $\mathbb{Z}$, generated by the loop γ above. Then for the function f above, what is the corresponding group homomorphism $\pi_1(f)$?*

Problem 6. *Let $X = S^1, Y = \mathbb{R}^2$ and let $f : X \to Y$ be the natural inclusion of the circle in the plane as the unit circle. What is $\pi_1(f)$?*