Simplicial Homology

Renzo’s math 571

Let us get our hands dirty with simplicial homology. There are two things I want to get you guys familiar with: one is how to compute simplicial homology. The other is to become familiar with homotopy in the category of \( \Delta \)-complexes, and see how one naturally obtains a homotopy of chain complexes for two homotopy equivalent \( \Delta \) complexes.

**Problem 1.** For the topological space assigned to your group:

1. Define a \( \Delta \)-complex structure.
2. Write down the chain complex, and the differential maps explicitly.
3. Compute all simplicial homology groups.

**Group 1** Torus;

**Group 2** \( \mathbb{R}P^2 \);

**Group 3** Klein Bottle;

**Group 4** Solid Torus.

**Problem 2.** For the pair of homotopy equivalent \( \Delta \)-complexes, construct explicitly homotopies that show that the corresponding chain complexes are homotopy equivalent.