1. If \( f(x, y, z) = \sqrt{x^3 + \sin(y) - y \ln(z)} \), find \( f(2, \pi/2, 1) \). Perform elementary simplifications.

**Solutions.** We evaluate by plugging the values for the \( x \), \( y \), and \( z \) coordinates:

\[
f(2, \pi/2, 1) = \sqrt{(2)^3 + \sin(\pi/2) - (\pi/2)\ln(1)} = \sqrt{8 + 1 - 0} = 3.
\]

\[\Box\]

2. Sketch the domain of \( g(x, y) = \ln(1 - 2x - 2y) \).

**Solution.** The domain of this function is determined by the equation

\[
1 - 2x - 2y > 0
\]

(since we only defined the natural log of positive numbers), and this inequality is the same as writing

\[
2y < 1 - 2x,
\]

\[
y < \frac{1}{2} - x
\]

an equation which describes the region below the line \( y = \frac{1}{2} - x \) in \( \mathbb{R}^2 \).

\[\Box\]

3. Let \( h(x, y, z) = 3x^2z + z \cos(\pi y - \pi x) + 3e^z \). Determine \( \lim_{(x,y,z) \to (1,2,0)} h(x, y, z) \).

**Solution.** We compute

\[
\lim_{(x,y,z) \to (1,2,0)} h(x, y, z) = h(1, 2, 0) = 3(1)^2(0) + (0)(\cos(\pi(2) - \pi(1))) + 3e^0
\]

\[
= 0 + 0 + 3 = 3,
\]

where the limit is the same as the value of the function since \( h \) is continuous. (That \( h \) is continuous follows from some comments about the continuity of its pieces as functions from \( \mathbb{R} \) to \( \mathbb{R} \). You are **not** required to show that \( h \) is a continuous function.)

\[\Box\]

4. The function \( k(x, y) = \frac{7x^8y}{-2x^9 + 9y^9} \) has no limit as \((x, y) \to (0, 0)\).

Show this by computing the limit of the function along the two following paths:

(a) \( t \mapsto (t, 0) \).
(b) $t \mapsto (t, t)$.

**Solution.** Consider the first path $t \mapsto (t, 0)$. We must compute the limit as $t \to 0$ of the expression $k(t, 0)$:

$$
\lim_{t \to 0} \frac{7t^8 \cdot 0}{-2t^9 + 9 \cdot 0^9} = \lim_{t \to 0} \frac{0}{-2t^9} = 0.
$$

For the second path $t \mapsto (t, t)$, we must substitute $x = t, y = t$:

$$
\lim_{t \to 0} \frac{7t^8 \cdot t}{-2t^9 + 9t^9} = \lim_{t \to 0} \frac{7t^9}{7t^9} = 1.
$$

Since $0 \neq 1$, meaning that two different paths toward the limit give different values, we conclude that

$$
\lim_{(x,y) \to (0,0)} \frac{7x^3y^3}{2x^6 + 9y^6}
$$

does not exist. 

---

5. Compute $\frac{\partial h}{\partial x}$ for the function in #3.

**Solution.** We compute

$$
\frac{\partial}{\partial x} (h(x, y, z)) =
$$

$$
\frac{\partial}{\partial x} (3x^2z + z \cos(\pi y - \pi x) + 3e^z) =
$$

$$
\frac{\partial}{\partial x} (3x^2z) + \frac{\partial}{\partial x} (z \cos(\pi y - \pi x)) + \frac{\partial}{\partial x} (3e^z) =
$$

$$
6xz + (-z \sin(\pi y - \pi x))(-\pi) + 0 =
$$

$$
6xz + \pi z \sin(\pi y - \pi x).
$$

Note that we **must** use the chain rule with $\cos(\pi y - \pi x)$ and that the partial derivative of $3e^z$ with respect to $x$ is equal to 0.