We will not answer questions about this page during the exam.

\( f \) is a real-valued function and \( \textbf{F}(x,y,z) = \langle M, N, P \rangle \) is vector-valued. (If in \( \mathbb{R}^2 \), \( \textbf{F} = \langle M, N \rangle \).) \( \textbf{T} \) is an appropriate unit tangent vector and \( \textbf{n} \) is an appropriate unit normal vector.

\( \textbf{r}(t) = \langle f(t), g(t), h(t) \rangle \) is a parameterization of a curve in \( \mathbb{R}^3 \) (\( \textbf{r}(t) = \langle f(t), g(t) \rangle \) in \( \mathbb{R}^2 \)); \( \textbf{r}(u,v) = \langle f(u,v), g(u,v), h(u,v) \rangle \) is a parameterization of a surface, with \( \textbf{r}_u = \frac{\partial \textbf{r}}{\partial u} \) and \( \textbf{r}_v = \frac{\partial \textbf{r}}{\partial v} \).

\[ \int_C f(x,y,z) \, ds = \int_a^b f(\textbf{r}(t))|\textbf{v}(t)| \, dt, \text{ where } \textbf{v}(t) = \textbf{r}'(t). \]

Line Integral along a curve \( C \):

Work/Circ/Flow along a curve \( C \):

- in \( \mathbb{R}^2 \): Work/Circ/Flow = \( \int_C \textbf{F} \cdot d\textbf{r} = \int_C M \, dx + N \, dy \). Also see Green’s Theorem.
- in \( \mathbb{R}^3 \): Work/Circ/Flow = \( \int_C \textbf{F} \cdot d\textbf{r} = \int_C M \, dx + N \, dy + P \, dz \). Also see Stokes’ Theorem.

Flux of vector field \( \textbf{F} \):

- across curve \( C \subset \mathbb{R}^2 \): \( \int_C \textbf{F} \cdot \textbf{n} \, ds = \int_C M \, dy - N \, dx \). Also see Green’s Theorem.
- through surface \( S \subset \mathbb{R}^3 \): \( \int_S \textbf{F} \cdot \textbf{n} \, d\sigma = \int_S \textbf{F} \cdot (\textbf{r}_u \times \textbf{r}_v) \, dudv \). Also see Divergence Theorem.

Component Test: \( \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} = \frac{\partial P}{\partial z} = \frac{\partial P}{\partial y} \). Equivalently, \( \text{curl}(\textbf{F}) = \nabla \times \textbf{F} = \textbf{0} \).

Fundamental Theorem for Line Integrals: If \( \textbf{F} = \nabla f \) and curve \( C \) goes from \( A \) to \( B \), then

\[ \int_C \textbf{F} \cdot d\textbf{r} = f(B) - f(A). \]

Green’s Theorem: Region \( R \subset \mathbb{R}^2 \) has closed boundary curve \( C \).

Work/Circ/Flow = \( \oint_C \textbf{F} \cdot d\textbf{r} = \int_C M \, dx + N \, dy + \int_R \left( \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) \, dx \, dy = \int_R \nabla \times \textbf{F} \cdot \textbf{k} \, dx \, dy \),

Flux = \( \oint_C \textbf{F} \cdot \textbf{n} \, ds = \int_C M \, dy - N \, dx + \int_R \left( \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} \right) \, dx \, dy = \int_R \nabla \cdot \textbf{F} \, dx \, dy \)

Surface Integral of \( g \) over the surface \( S \) (\( g(x,y,z) = 1 \) for surface area):

\[ \int_S g(x,y,z) \, d\sigma = \int_R g(\textbf{r}(u,v))|\textbf{r}_u \times \textbf{r}_v| \, dudv, \text{ with parameters } u,v \text{ in } R. \]

Stokes’ Theorem: Surface \( S \) with closed boundary curve \( C \) (where \( C \) has counterclockwise orientation with respect to the normal direction of \( S \)).

Work/Circ/Flow = \( \iint_S \textbf{F} \cdot d\textbf{r} = \int_C M \, dx + N \, dy + P \, dz + \int_S \nabla \times \textbf{F} \cdot d\sigma = \iint_S \nabla \times \textbf{F}(u,v) \cdot (\textbf{r}_u \times \textbf{r}_v) \, dudv \).

Divergence Theorem: Solid \( D \) with boundary surface \( S \).

Flux = \( \iiint_S \textbf{F} \cdot \textbf{n} \, d\sigma = \int_S \textbf{F}(\textbf{r}(u,v)) \cdot (\textbf{r}_u \times \textbf{r}_v) \, dudv = \iint_D \nabla \cdot \textbf{F} \, dV = \iiint_D \left( \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z} \right) \, dV. \)