Math 281
Homework
Oct. 24th

1. If \( g, h \in G \) (note: the operation is not necessarily commutative) are group elements, what is the inverse of the element \( gh \) in terms of \( g^{-1} \) and \( h^{-1} \)?

2. For each of the following functions, prove or disprove that it is a group homomorphism:

(a) \[ f : \mathbb{Z}/3\mathbb{Z} \rightarrow \mathbb{Z}/6\mathbb{Z} \]
   \[ [x] \mapsto [4x] \]

(b) \[ f : \mathbb{Z}/3\mathbb{Z} \rightarrow \mathbb{Z}/6\mathbb{Z} \]
   \[ [x] \mapsto [3x] \]

(c) \[ f : S_2 \rightarrow S_3 \]
   \[ e \mapsto e \]
   \[ (12) \mapsto (23) \]

(d) \[ f : S_2 \rightarrow S_3 \]
   \[ e \mapsto e \]
   \[ (12) \mapsto (123) \]

(e) \[ f : \mathbb{Z} \rightarrow S_5 \]
   \[ n \mapsto (1245)^n \]

3. For any of the above that are homomorphisms, write down the kernel and the image.

4. Let \( \phi : G \rightarrow H \) be a group homomorphism. If \( g \in G \) is an element of order \( k \), what can you say about the order of \( \phi(g) \)? (Remember to look at the various examples of group homomorphisms and functions that are not group homomorphisms in order to understand what is going on!)

5. Let \( A = (12)(34) \), \( B = (13)(24) \), \( C = (14)(23) \) be elements in \( S_4 \). Prove that the function below is a group homomorphism. Have you seen it before?
\[
\phi : S_4 \to S_3 \\
g \mapsto \begin{cases} 
A &\mapsto g^{-1}Ag \\
B &\mapsto g^{-1}Bg \\
C &\mapsto g^{-1}Cg 
\end{cases}
\]