

Recovery Homework 1: Sets and Equivalence Relations

Math 281

1. Write down some examples of sets whose elements are NOT numbers. In particular, give me three examples of finite sets, one example of an infinite but countable set, and one example of an infinite uncountable set. For each of the sets you gave, give me a finite subset and (if possible) an infinite subset.
2. Consider the sets:
 - $A = \{Bob, Joe, Jane, 1, 5, 7\}$
 - $B = \{Mark, Joe, 2, 5, one\ potato, one\ carrot\}$
 - (a) How many elements does A have?
 - (b) How many elements does B have?
 - (c) What is the set $A \cup B$? And $A \cap B$?
 - (d) Can you write down an equation relating the number of elements for $A, B, A \cap B, A \cup B$? Is your equation specific to this example, or does it hold true for any pair of sets? If it does, prove it!

Now let me give you the following subsets of $A \cup B$:

- $C_1 = \{Bob, Joe, Jane, Mark\}$
 - $C_2 = \{1, 2, 5, 7\}$
 - $C_3 = \{one\ potato, one\ carrot\}$
- (a) Do these subset determine an equivalence relation on $A \cup B$?
 - (b) Can you describe such an equivalence relation in words? (i.e. with a sentence of the form " $a \sim b$ if...")?
 - (c) How many elements does the quotient set have? What are the elements of your quotient set?
 - (d) Write a bubble diagram of $A \cup B$ on the left, and of the quotient set $A \cup B / \sim$ on the right, and draw arrows to represent the quotient function.

Now let me give you the following subsets of $A \cup B$:

- $C_1 = \{Bob, 2, Jane, Mark\}$
- $C_2 = \{1, one\ carrot\}$
- $C_3 = \{one\ potato, 7\}$
- $C_4 = \{Joe, 5\}$

- Do these subset determine an equivalence relation on $A \cup B$?
- Can you describe such an equivalence relation in words? (i.e. with a sentence of the form “ $a \sim b$ if...?” (I can’t! Not all equivalence relations need to be described in sentences!!))
- How many elements does the quotient set have? What are the elements of your quotient set?
- Write a bubble diagram of $A \cup B$ on the left, and of the quotient set $A \cup B / \sim$ on the right, and draw arrows to represent the quotient function.

Finally let me give you the following subsets of $A \cup B$:

- $C_1 = \{1, 2, Bob, Joe, Jane, Mark\}$
- $C_2 = \{1, 2, Bob, 7\}$
- $C_3 = \{one\ carrot\}$

- Can the above subsets be the equivalence classes for an equivalence relation on $A \cup B$? Why or why not?

- Let $X = \{1, 2, \dots, 10\}$ be the set of the first 10 natural numbers. For each of the following “laws of squiggling”, decide if it is an equivalence relation. If it is, write down the equivalence classes, and describe the quotient set. In particular, say how many elements does the quotient set have.

- $a \sim b$ if $b - a$ is an integer multiple of 5;
- $a \sim b$ if $b - a$ is an integer multiple of 3 AND of 2;
- $a \sim b$ if $b - a$ is an integer multiple of 3 OR of 2;
- $a \sim b$ if $b - a$ is an integer multiple of 5, or if a and b both belong to the subset $\{1, 2, 6, 7\}$;