HOMEWORK ON EULER CHARACTERISTIC FOR SURFACES

This worksheet accompanies today's lecture on Euler characteristic for surfaces. The following problems are designed to lead to the discovery of the Euler characteristic and to the understanding of why it is a topological invariant.

Problem 1 *Count the number of vertices (V), edges (E) and faces (F) for:*

- *a tetrahedron;*
- *a cube;*
- *a polyhedron of your choice.*

Now compute the following sum:

$$V - E + F$$

What happens?

In topology all polyhedra are just **spheres**, and we can think of edges and vertices as a graph on the sphere.

Hope: maybe the above number does not depend on the graph you put on a sphere!

Problem 2 *Show that the above hope is too much to ask for. Find (simple) graphs on the sphere for which the above count is not* 2*.*

Fix: A **good graph** (on ANY surface, not just a sphere) is a graph such that:

1. there is a vertex any time two edges intersect.
2. there is a vertex at each end of every edge.
3. the complement of the graph is homeomorphic to a disjoint union of discs.

Problem 3 *Put a good graph on a surface. What happens to the count*

$$V - E + F$$

when:

- *you add a vertex in the middle of an edge?*
- *you add an edge connecting two vertices?*
- *you add a vertex in the middle of a face and an edge to connect to an existing vertex?*

Use this to prove that this count is independent of the good graph you put on the surface.

We have just defined a **topological invariant**, called **Euler characteristic**. Now let us use it !

Problem 4 *What is the Euler characteristic of:*

- *the torus (doughnut)?*
- *the g-tours (doughnut with g holes)?*