

676: Algebraic Number Theory, spring 2007

Major Goals:

1. Gauss: quadratic reciprocity - solutions to degree two equations modulo p .
2. Kronecker-Weber: classifying abelian extensions of \mathbb{Q} .
3. Kummer: proof of Fermat's last theorem for regular primes.
4. Dirichlet: The proportion of primes in each congruence class modulo p .
5. Hasse-Weil: Counting the number of points on an elliptic curve over a finite field.

Tentative syllabus:

1. Crucial examples
 - * finite fields [IR7]
 - * quadratic and cyclotomic fields [IR13] [J1.9-1.10]
 - * quadratic reciprocity [EM7] [J1.11]
2. Rings of algebraic integers and unique factorization [N1] [J1] [EM4-6]
 - * integrality [S5]
 - * discriminant [S10]
 - * factorization of ideals [S6], [IR12]
 - * finiteness of class group [S11]
 - * decomposition and inertia groups [S13,14] [J3]
3. Zeta functions
 - * Riemann-zeta function, Dirichlet's theorem [IR16] [R22-25] [N7] [J4]
 - * curves over finite fields [IR8,11]
 - * Weil conjectures for elliptic curves [Sil5]

Books: We will talk about textbook choices in the first week of class.

- [S] Stein: A brief introduction to classical and algebraic number theory; (straight-forward, also has algorithms). free at <http://modular.math.washington.edu/papers/ant/>
- [J] Janusz, Algebraic Number Fields 2nd edition, Graduate Studies in Mathematics, Volume 7, AMS \$45 (local approach, clearly written, no exercises).
- [IR] A Classical Approach to Modern Number Theory, 2nd edition, Ireland-Rosen, Springer-Verlag \$70 (clearly written, contains more background material, focus on Gauss sums, good problems but not many examples)
- [R] Ribenboim, Classical Theory of Algebraic Numbers, \$80. (covers everything, many examples, long-winded, overly detailed)

[EM] Esmonde and Murty, Problems in Algebraic Number Theory, Springer \$60. (contains many problems and their solutions).

[N] Algebraic Number Theory, Neukirch, Springer \$120 (beautifully written, more sophisticated approach, not many examples).

[Sil] Silverman, The arithmetic of Elliptic curves, Springer-Verlag.

and... Lang (class field theory), Marcus (great examples, not type-set), Koch.