Pries: 619 Complex Variables II. Homework 6.

1. Let $g(z) = f'(z)/f(z)$. If $f(z)$ has a pole of order k at w, show that $\mathrm{res}_w(g) = -k$.
2. Let $m(z) = zf'(z)/f(z)$. If $f(z)$ has a pole of order k at w, show that $\mathrm{res}_w(m) = -kw$.
3. Find out more about the winding number and its relevance to the proof of Prop. C.
4. Let $f(z) = 4P(z)^3 - 60G_4P(z) - 140G_6$.
   i) Show that $P'(z)^2 - f(z)$ is holomorphic near $z = 0$ and has a zero at $z = 0$ by looking at its Laurent series.
   ii) Explain why $P'(z)^2 - f(z)$ is holomorphic everywhere.
   iii) Explain why $P'(z)^2 - f(z)$ is the zero-function.
   iv) Explain why X has an equation of the form $y^2 = f(z)$ where $f(z)$ has degree 3.
5. As in the lecture, let $f(z) = 4P(z)^3 - 60G_4P(z) - 140G_6$ and let $\tau : \mathbb{C}/L \to E$. Show that E is non-singular.
6. Silverman VI.6.6