Pries: 619 Complex Variables II. Homework 1.

1. Pick one unfamiliar word or historical reference from the first lecture. Investigate it and give a SHORT summary.

2. Miranda: I.1.F

3. Check that the two stereographic projection maps are compatible with the transition map: $\phi_1 = T \circ \phi_2$ (This is Miranda I.1.G).

4. Show that the reflection ρ_y in S^2 over the plane $y = 0$ corresponds (via stereographic projection ϕ_1) to complex conjugation cc in $\mathbb{C}_\infty$: $cc \circ \phi_1 = \phi_1 \circ \rho_y$.

5. Show that the 1-point compactification of the real line is homeomorphic to the circle.

6. A. The eight points with coordinates $(\pm 1/\sqrt{3}, \pm 1/\sqrt{3}, \pm 1/\sqrt{3})$ form the vertices of a cube inside the sphere. Find the images of these points under stereographic projection.

   B. The five platonic solids correspond to the five regular tilings of the sphere. We can suppose that the center of one face is at $(0, 0, 1)$. The image of these five tilings under stereographic projection correspond to five graphs in the complex plane. What adjectives describe the combinatorial properties of these graphs? (Ask Anton!)

   C. Give a heuristic explanation why the groups A_4, S_4, and A_5 are subgroups of the automorphism group of $\mathbb{C}_\infty$.