Pries, Spring 2019, Algebraic Number Theory
Week 6:

Key concepts:

1. Finite fields
2. Unique factorization
3. Fractional ideals and class groups

Reading List:

1. Ireland-Rosen, Sections 7.1-7.2, 12.2-12.3, 13.1-13.2.
2. Next up: Marcus chapters 3-5.

Homework: Do either problems (A)-(B) or problems (B)-(C). Due Fri 3/8.
Ireland-Rosen:
Chapter 7 (A) #6,16 (B) #3, 15, (C) #17.
Chapter 13 (A) #1, 6, (B) #8, 18, (C) #19
Let $\mathbb{F}_q$ denote the finite field of size $q = p^a$. By FLT, $\alpha^q = \alpha$ for all $\alpha \in \mathbb{F}_q$.

1. If $\alpha \in \mathbb{F}_q$, prove that the minimal polynomial $f(x)$ for α over $\mathbb{F}_p$ divides $x^q - x$.
2. Prove that $\mathbb{F}_{p^a} \subset \mathbb{F}_{p^b}$ if and only if $a \mid b$.
3. Hand in your data and observations about one of the SAGE problems.

Pries, Spring 2019, Algebraic Number Theory
Week 4:

Key concepts:

1. Integral bases
2. Quadratic reciprocity
3. Gauss sums

Reading List:

1. Ireland-Rosen, Sections 5.1-5.2, 6.2-6.3, 7.3.

Homework 3: Do either problems (A)-(B) or problems (B)-(C). Due Fri 2/22.
Ireland-Rosen:
Chapter 5: (A) #2,4,11,16, (B) #7,14,15, (C) page 86 #7,8
Chapter 6: (A) #9,10, (B) #8, (C) #11

Pries, Spring 2019, Algebraic Number Theory
Week 3:

Key concepts:

1. Number fields and integral closure
2. The discriminant

Reading List: Try to read three of these references.

1. Marcus, Chapter 2.
2. Ogglier (short), Section 1.2.
3. Ireland-Rosen, Section 1.3-1.4, Section 12.1.
4. Milne (harder), Chapter 2 pages 30-36.

Theorem of the week: We will discuss this theorem on Monday 2/4.

1. Marcus, pages 11-12, Theorem 2 and Corollary 1.
2. Ogglier, page 9, Theorem 1.3 and Corollary 1.4.
3. Ireland-Rosen, page 68, Proposition 6.1.5.
4. Milne, page 25, Theorem 2.1.

Homework 2: Do either problems (A)-(B) or problems (B)-(C). Due Fri 2/8.
Marcus, Chapter 2: pages 28-38, (A) 1b, 7, 8b, 31 (B) 15, 16, 33, (C) 8ac, 35abcd.

Pries, Spring 2019, Algebraic Number Theory
Week 1: A road map to get started.

Key concepts:

1. Quadratic fields and rings
2. Number fields and integral closure
3. Motivation from Fermat's Last Theorem

Reading List: Try to read three of these references.

1. Marcus, Chapter 1, Chapter 2 pages 9-12.
2. Ogglier (short), Section 1.1.
3. Ireland-Rosen, Section 1.1-1.2 (E), Section 6.1.
4. Milne (harder), Introduction pages 7-12, Chapter 2 pages 25-30.

Theorem of the week: We will discuss this theorem on Monday 1/28.

1. Marcus, pages 11-12, Theorem 2 and Corollary 1.
2. Ogglier, page 9, Theorem 1.3 and Corollary 1.4.
3. Ireland-Rosen, page 68, Proposition 6.1.5.
4. Milne, page 25, Theorem 2.1.

Homework 1: Do either problems (A)-(B) or problems (B)-(C). Due Wed 1/30.
Marcus, Chapter 1: pages 4-6, problems (A) 1-6, (B) 10,12,13, (C) 16,17,18.