
Pries: 466 Abstract Algebra I:
Midterm 1, September 25, 2017.

Name:________________________

Short questions: (5 problems worth 4 points each). Give 1-2 sentences justification.

1. What is the center of D_{12}?

2. Find an element $g \in S_5$ such that $g(1,2,3)g^{-1} = (3,5,2)$.

3. The group G of symmetries of the cube acts on the set of pairs of opposite faces of the cube. How big is the stabilizer of the pair FB (front and back face)?

4. The group $G = (\mathbb{Z}/11)^*$ acts on $A = (\mathbb{Z}/11)^*$ by $g \cdot a = g^2 * a$. Is this action faithful? Explain why or why not.

5. The group $\mathrm{GL}_2(\mathbb{R})$ acts on $\mathbb{R}^2 - \{(0,0)\}$ by $M \cdot (x,y) = (\det(M)x, \det(M)y)$. Is this action transitive? Explain why or why not.

Long questions: (4 problems worth 10 points each). Provide a complete solution and show all your work.

1. If G acts on A and $g \cdot a = b$, prove that $g\mathrm{St}_a g^{-1} = \mathrm{St}_b$.

2. The group D_8 acts on the set of opposite vertices of the octagon, which is labeled as $A = \{\bar{15}, \bar{26}, \bar{37}, \bar{48}\}$. The stabilizer of $\bar{15}$ is $S = \{1, f, r^4, r^4 f\}$. Find the left cosets of S in D_8. Match up the left cosets with the orbit of $\bar{15}$ in the way used in the proof of the Orbit-Stabilizer Theorem.

3. If G acts on a set A, the fundamental theorem says that there is a homomorphism $\phi : G \to S_A$. Let $K = \{g \in G \mid g \cdot a = a \text{ for all } a \in A\}$. Prove that $K = \mathrm{Ker}(\phi)$.

4. Prove that the symmetry group of the cube is S_4.

5. Please copy the honor pledge and sign your name: "I have not given, received, or used any unauthorized assistance."