Pries: M460 - Information and Coding Theory, Spring 2019
Handout 3W: Hamming code

Group work The extended Hamming (7,4) code has this generator matrix.

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

Problems

1. Give a formula for the value p_4 in the last column in terms of d_1, d_2, d_3, d_4.
2. Explain how to compute p_4 using parity and explain this with a Venn diagram picture.
3. What are the length n and the dimension k of the extended Hamming $(7, 4)$ code?
   What is the information rate k/n?
   Is it more or less than the information rate of the Hamming $(7, 4)$ code?
4. What is the minimum distance d of the extended Hamming $(7, 4)$ code?
   What is the relative error correction rate d/n?
   Is it more or less than the relative error correction rate of the Hamming $(7, 4)$ code?
5. How many errors can this code detect? How many errors can this code correct?

Pries: M460 - Information and Coding Theory, Spring 2019
Homework: Due Friday 2/22

Here is another way of constructing the Hamming $(7, 4)$ code.

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 10 & 11 & 100 & 101 & 110 & 111 \\ p_1 & p_2 & d_1 & p_3 & d_2 & d_3 & d_4 \\ X & 0 & X & 0 & X & 0 & X \\ 0 & X & X & 0 & 0 & X & X \\ 0 & 0 & 0 & X & X & X & X \end{pmatrix}$$

The last 3 rows give the relations

$$p_1 = d_1 + d_2 + d_4,\ p_2 = d_1 + d_3 + d_4,\ p_3 = d_2 + d_3 + d_4.$$

To find the 4×7 generator matrix, take the 4×4 identity matrix and adjoin the 4×3 matrix which is the transpose of the data columns. **Problems**

1. Find the relations on p_1 and p_2 given by just the first three columns. What code from class is this the same as?
2. Construct a similar table for $n = 15$.

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \\ 1 & 10 & 11 & 100 & 101 & 110 & 111 & & & & & & & & \\ p_1 & p_2 & d_1 & p_3 & d_2 & d_3 & d_4 & & & & & & & & \\ X & 0 & X & 0 & X & 0 & X & & & & & & & & \\ 0 & X & X & 0 & 0 & X & X & & & & & & & & \\ 0 & 0 & 0 & X & X & X & X & & & & & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & & & & & & & & \end{pmatrix}$$

3. Find the 4 relations for this code:

$$p_1 =$$

$$p_2 =$$

$$p_3 =$$

$$p_4 =$$

4. What are the invariants: length $n = 15$, dimension k, info rate k/n for this code?
5. What is the minimum distance d? If you're not sure, find the generator matrix.
6. Extra Credit: It is possible to construct a Hamming code $H(n, k)$ in this way for any length $n = 2^m - 1$. How many parity check digits will it have? Find a formula for its dimension k and its minimum distance d in terms of m.