Pries: M460 - Information and Coding Theory, Spring 2019
Handout 1F: the ISBN error-correcting code

An *ISBN number* is a ten digit string $a = a_1 - a_2a_3a_4 - a_5a_6a_7a_8a_9 - a_{10}$, with each digit in $\mathbb{Z}/11\mathbb{Z} = \{0, 1, \ldots, 9, X\}$ such that

$$a_{10} \equiv 1 \cdot a_1 + 2 \cdot a_2 \cdots 9 \cdot a_9 \bmod 11, \text{ or, equivalently, } \sum_{i=1}^{10} i \cdot a_i \equiv 0 \bmod 11.$$

1. What is the check digit a_{10} for the book with ISBN $1 - 111 - 11111 - a_{10}$?

2. How many ISBN numbers are there?

3. What is the information rate of the ISBN code?

4. Find all ISBN numbers of the form $a_1 - 000 - 00000 - a_{10}$.

5. How many errors can the ISBN code detect? How many can it correct?

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Homework 2: the ISBN error-correcting code
Due Friday 2/1

In class we showed that the ISBN code can detect one error. Suppose that, for one i with $1 \leq i \leq 10$, the value a_i is replaced by $b_i = a_i + \epsilon$ ($\epsilon \not\equiv 0 \bmod 11$). Then the left hand side of the relation $\sum_{i=1}^{10} i \cdot a_i \equiv 0 \bmod 11$ changes by $i\epsilon \not\equiv 0 \bmod 11$.

1. A librarian wants to type an ISBN number $a = a_1 - a_2a_3a_4 - a_5a_6a_7a_8a_9a_{10}$. By mistake, the librarian transposes two non-equal digits. (For some $i \neq j$, for which $a_i \neq a_j$, the librarian types $b_i = a_j$ and $b_j = a_i$.) Explain why the formula for the check digit will produce an error.

2. Suppose a and b are ISBN numbers and $0 \leq \kappa \leq 10$. If $d_i = a_i + \kappa b_i$ for $1 \leq i \leq 10$, show that $d = d_1 - d_2d_3d_4 - d_5d_6d_7d_8d_9 - d_{10}$ is also an ISBN number.
   Remark: this shows that the set of ISBN numbers is a *vector space* over $\mathbb{Z}/11\mathbb{Z}$.

3. Here are 9 ISBN numbers:

   $$v_1 = 1 - 000 - 00000 - 1,\ v_2 = 0 - 100 - 00000 - 2,\ \ldots, v_9 = 0 - 000 - 00001 - 9.$$

   Show that $\{v_1, \ldots, v_9\}$ is a *basis* for the set of ISBN numbers as follows:.

   (a) Span: If a is an ISBN number, explain why there are constants $c_1, \ldots, c_9 \in \mathbb{Z}/11\mathbb{Z}$ such that $a = c_1v_1 + \cdots + c_9v_9$.

   (b) Linear Independence: If $c_1, \ldots, c_9 \in \mathbb{Z}/11\mathbb{Z}$ are constants such that $c_1v_1 + \cdots c_9v_9 = 0 - 000 - 00000 - 0$, explain why $c_1, \ldots, c_9$ must all be zero.