

## Pries: M467 - Information and Coding Theory, Spring 2014

**Read:** Roman Chapter 3

### Problems

1. Roman 3.4 #1,2. Suppose the source  $S = \{a, b, c\}$  has probability distribution  $P = \{p(a) = 1/4, p(b) = 1/4, p(c) = 1/2\}$ . Is it possible to construct an instantaneous code whose average codeword length equals the binary entropy? What if  $P = \{p(a) = 1/6, p(b) = 1/6, p(c) = 2/3\}$ ?
2. Roman 3.1 #7. Suppose we toss a fair coin and roll a fair die. Do we get more information (on average) from this experiment or from the experiment of tossing 3 fair coins? Four fair coins?
3. Roman 3.1 #8. How much information do we get (on average) by sampling from a deck of cards if:
  - a) each card is equally likely to be drawn?
  - b) the black cards are twice as likely to be drawn as the red cards?
4. Roman 3.1 #9. Suppose we roll a fair die that has two faces numbered 1, two faces numbered 2, and two faces numbered 3. Then we toss a fair coin the number of times indicated by the number on the die and count the number of heads. How much information is obtained (on average) from this procedure?
5. How many source words do there need to be to force every uniquely decipherable binary encoding to have average word length at least 5?
6. Extra credit: Roman 3.2 #6. Let  $P = \{p_1, \dots, p_n\}$  be a probability distribution. Suppose  $\epsilon > 0$  and  $p_1 - \epsilon > p_2 + \epsilon > 0$ . Thus  $\{p_1 - \epsilon, p_2 + \epsilon, p_3, \dots, p_n\}$  is also a probability distribution. Explain in words and then prove the inequality:

$$H(p_1, \dots, p_n) < H(p_1 - \epsilon, p_2 + \epsilon, p_3, \dots, p_n).$$