Pries: 405 Number Theory: Spring 2020: homework

| Day | Assignment |
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| $1 / 24$ | Read: Stillwell 1.6 |
|  | Vocab: what is a Diophantine equation? |
|  | Warm-up: 1.6.3-1.6.4 |
| $1 / 27$ | Read: Stillwell 1.7 |
|  | Vocab: what is a rational point? |
|  | Warm-up: 1.7.1-1.7.4 |
| $1 / 29$ | Read: Stillwell 1.8 |
|  | Vocab: what is a Gaussian integer? |
|  | Warm-up: 1.8.4-1.8.6 |
| $1 / 31$ | Read: Stillwell 2.1-2.2 |
|  | Vocab: what is a gcd? (of integers, polynomials, Gaussian integers) |
|  | Warm-up: 2.1.1-2.1.2, 2.2.1-2.2.2 |
| $2 / 3$ | Read: Stillwell 2.3 |
|  | Vocab: What is the Euclidean algorithm? (of integers, polynomials, Gaussian integers) |
|  | Warm-up: 2.3.1 |
| $2 / 5$ | Read: Stillwell 2.4 |
|  | Vocab: What is unique factorization (of integers, polynomials, Gaussian integers) |
|  | Warm-up: Carefully review the proof of the prime divisor property |
| $2 / 7$ | Read: Stillwell 2.5 |
|  | Vocab: What does it mean to be prime? irreducible? |
|  | Warm-up: Redo the irrational square roots proof for cube roots |
| Due $2 / 7$ | Homework assignment 1 |
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See next page.

| Day | Assignment |
| :--- | :--- |
| $2 / 10$ | Read: Stillwell 2.6, 5.1 |
|  | Vocab: What is Pell's equation? |
|  | Warm-up: 2.6.1, 2.6.2 |
| $2 / 12$ | Read: Stillwell 5.2, 5.3 |
|  | Vocab: What does it mean to have a group of solutions? |
|  | Warm-up: 5.3.1-5.3.3 |
| $2 / 14$ | Read: Stillwell 5.4, 5.5 |
|  | Vocab: Explain the connection between solutions to Pell's equation and units in $\mathbb{Z}[\sqrt{n}]$ |
|  | Warm-up: 5.4.4, 5.4.5 |
| $2 / 17$ | Read: Stillwell 6.1, 6.2 |
|  | Vocab: What is a prime in $\mathbb{Z}[i]$ ? |
|  | Warm-up: $6.2 .3,6.2 .4$ |
| $2 / 19$ | Read: Stillwell 6.3, 6.4 |
|  | Vocab: Why does $\mathbb{Z}[i]$ have a Euclidean algorithm? |
|  | Warm-up: $6.3 .4,6.3 .5,6.3 .6$ |
| $2 / 21$ | Midterm 1 |
|  |  |

See next page.

If you feel like class is going too fast, I encourage you to spend some extra time reading. Good sections to read are: 7.1-7.4, Chapter 10.
Write down a new thing you learned from each section.

| Day | Assignment |
| :--- | :--- |
| $3 / 2$ | Read: Stillwell 11.1-11.3 |
|  | Vocab: Why doesn't $\mathbb{Z}[\sqrt{-6}]$ have a Euclidean algorithm? |
|  | Warm-up: 11.3.1-11.3.3 |
| $3 / 4$ | Read: Stillwell 11.4-11.5 |
|  | Vocab: Explain why a principal ideal in $\mathbb{Z}[\sqrt{-6}]$ needs to have the same shape as $\mathbb{Z}[\sqrt{-6}]$. |
|  | Find a non-principal ideal in $\mathbb{Z}[\sqrt{-6}]$. |
|  | Warm-up: 11.5.2-11.5.3 |
| $3 / 6$ | Read: Stillwell 11.6-11.7 |
|  | Vocab: Every ideal is a lattice. Is every lattice an ideal? |
| $3 / 9$ | Warm-up: 11.7.1-11.7.3 |
|  | Read: Stillwell 11.8-11.9 |
|  | Vocab: what is the difference between unique factorization of elements and of ideals? |
|  | Warm-up: Find an example of failure of unique factorization in $\mathbb{Z}[\sqrt{-6}]$ and fix it using non |
| $3 / 13$ | Vocab: What is the main theorem connecting $R / I$ to $I$ being prime? being maximal? |
|  | Read: Stillwell 12.5-12.7 |
| Due 3/13 | Vocab: What is a class group? |

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M405 Number Theory: Pries
Plans for 2 weeks after spring break

| Day | Assignment |
| :--- | :--- |
| $3 / 23$ | Read: project choices handout |
| Due $3 / 23$ | 3 project choices (labeled first, second, third) <br> Submit by e-mailing me: rachelpries@gmail.com |
| $3 / 25$ | Read: Stillwell 5.6 |
|  | Vocab: What is a quadratic form? |
| $3 / 27$ | Warm-up: 5.6.1-5.6.3 <br> The link is at www.math.colostate.edu/ pries/405/405spring20/405hwsp20.html |
| Due $3 / 27$ | Computer lab: <br> Week 8 Monday, continued fractions, problems 3,4,5 <br> Week 8 Wednesday, cyclotomic fields, problems 3,4,5, extra credit 6 <br> Week 8 Friday, elliptic curves, problems 2,3,4 <br> Submit by e-mailing me, preferably pdf file. |
| $3 / 30$ | Read Stillwell 8.1, 8,2, 8.3. |
|  | Vocab: Why are quaternions good for studying sums of 4 squares? |
| $4 / 1$ | Warm-up: 8.2.3, 8.3.3 |
|  | Read Stillwell 8.4, 8.5, 8.6 |
| $4 / 3$ | Vocab: How are the Hurwitz integers different from $\mathbb{Z}[i, j, k]$ |
|  | Warm-up: 8.4.1, 8.5.1, 8.6.1, 8.6.3 |
|  | Voad Stillwell 8.7, 8.8, 8.9 |
| Due $4 / 3$ | Wourm- What is the 4-square theorem? What is a number that is not a sum of 3 squares? <br> Submit by e-mailing me pdf file |

Guidelist list: these will be developed more later but you need
Topic
Motivation for Topic
Definitions and Notation
Main Theorem
Interesting examples
Data/graphs/pictures
Sources: at least 3
In the long run, this material will be put in Poster format

