Pries: M405 - Number Theory, Spring 2018

Week 8 Monday: continued fractions in SAGE

1. Getting started:
   (a) Method 1: go to http://sagecell.sagemath.org/ and type 2+3 then evaluate
   (b) Method 2: (required for bigger jobs) start a free CoCalc account at http://www.sagemath.org/
   (c) http://doc.sagemath.org/html/en/reference/, quick search for continued fraction
   (d) Open and skim sage.rings.continued_fraction
2. Explain the output of these commands:
   (a) `gcd(97,100)`
   (b) `Mod(97, 100)^(-1)`
   (c) `xgcd(97,100)`
   (d) `plot((1-97*x)/100, -.1, .1) \\`
   (then change -.1, .1 to values a,b that illustrates the output of (c))
   (e) `c=continued_fraction(100/97); c`
   (f) `c.convergents();`
3. What pattern do you see in (a) and how does it compare with (b)-(c)?
   (a) c=continued_fraction([1,1,1,1,1,1,1,1]); c.convergents;
   (b) `K.<sqrt5> = QuadraticField(5); d=(1+sqrt5)/2; d.continued_fraction();`
   (c) `N(c.value() - d);`

4. For $p = 2, 3, 5, 7, 11$, do:

   (a) $N(sqrt(p))$;

   (b)
   ```
   continued_fraction(sqrt(p)) or
   ```
   ```
   continued_fraction_list(sqrt(p), nterms=20)
   ```
   What kind of behavior do you notice?

   (c) What is different for the output of:
   ```
   K.<sqrt2> = QuadraticField(2); cf=continued_fraction(sqrt2); cf;
   ```

   (d) What do these commands do and what are they for $p = 2, 3, 5, 7, 11$?
   ```
   cf.period(); cf.preperiod();
   ```

5. Conceptual problems:

   (a) If α is a fraction, explain why the continued fraction of α terminates.

   (b) If the continued fraction of α terminates, explain why α is a fraction.

   (c) If α has continued fraction $[\overline{1, 2}]$, find a quadratic polynomial which has α as a root. What is α?