NAME: __________________________
SI: ____________________________

SECTION NUMBER: _____

You may NOT use calculators or any references. Show work to receive full credit.

GOOD LUCK !!!
1. Consider the double integral in the Cartesian coordinates
\[ \int_0^6 \int_0^y x \, dx \, dy. \]
Change the integral into an equivalent polar integral. **Do not evaluate.**

2. Using **Cylindrical** coordinates set up, **but do not evaluate**, the triple integral
\[ \iiint_E x y z \, dV, \]
where \( E \) is the circular cylinder whose base is the circle \( x^2 + (y - 1)^2 = 1 \) and whose top lies in the plane \( z = 5 - x \).

3. Consider the triple integral \( \iiint_D x \, dV \), where \( D \) is the wedge cut from the cylinder \( x^2 + y^2 = 1 \) by the planes \( z = -y \) and \( z = 0 \) in the half-space \( z \geq 0 \).
   
   (a) Sketch the projection of \( D \) onto the plane you use to set up the triple integral. Label the sketch appropriately.
   
   (b) **Set up, do not evaluate**, this integral in **Cartesian (Rectangular)** coordinates.
   
   (c) **Set up, do not evaluate**, the triple integral in **Cylindrical** coordinates.
   
   (d) Evaluate the integral you set up in (c).

4. Consider the solid \( E \) in the first octant bounded between the planes \( x + y + 2z = 2 \) and \( 2x + 2y + z = 4 \) with density \( \delta(x, y, z) = xyz \).
   
   (a) Sketch \( E \) and label appropriately.
   
   (b) **Set up, do not evaluate**, a triple integral that computes the **volume** of \( E \). Be sure to include a sketch of the projection.
   
   (c) **Set up, do not evaluate**, a triple integral that computes the **mass** of the solid \( E \).

5. Set up, do not evaluate, the volume of the smaller region cut from the solid sphere \( \rho \leq 2 \) by the plane \( z = \sqrt{3} \) in the first octant in **Spherical** coordinates.

6. Use the change of variable equations \( x = 3r \cos \theta, \ y = 2r \sin \theta \) to find the volume of the region bounded by the \( xy \)-plane, the paraboloid \( z = x^2 + y^2 \), and the elliptical cylinder \( \frac{x^2}{9} + \frac{y^2}{4} = 1 \). The half-angle formulas are useful:
\[ \cos^2 \theta = \frac{1 + \cos 2\theta}{2} \quad \text{and} \quad \sin^2 \theta = \frac{1 - \cos 2\theta}{2}. \]
7. Let \( C = C_1 \cup C_2 \cup C_3 \), where \( C_1 : y = x^2 \) from \((0,0)\) to \((2,4)\); \( C_2 \) : line segment joining \((2,4)\) and \((2,0)\); \( C_3 \) : line segment joining \((2,0)\) and \((0,0)\). For the velocity field \( \mathbf{F} = xi - yj \), find

(a) The flow of \( \mathbf{F} \) along \( C_1 \).

(b) The outward flux of \( \mathbf{F} \) across the closed path \( C \) if the outward flux of \( \mathbf{F} \) over \( C_2 \) is \(-8\) and the outward flux of \( \mathbf{F} \) over \( C_3 \) is \(0\).

8. Consider the vector field \( \mathbf{F}(x, y, z) = \langle 2xz, \sin z, x^2 + y \cos z \rangle \).

(a) Show that \( \mathbf{F} \) is conservative.

(b) Evaluate the line integral \( \int_C \mathbf{F} \cdot d\mathbf{r} \), where \( C \) is the line path that connects \( A(1, 2, \pi) \) to \( B(e, 4, 0) \), \( B \) to \( C(1, 2, -3) \), and \( C \) to \( D(2,1,2\pi) \).