MATH261 EXAM III SPRING 2008

NAME:____________________________
SI:_______________________________

SECTION NUMBER: _____

You may NOT use calculators or any references. Show work to receive full credit.

GOOD LUCK !!!

Problem	Points	Score
1	10	
2	10	
3	15	
4	10	
5	10	
6	15	
7	15	
8	15	
Total	100	

1. Consider the double integral in the Cartesian coordinates

$$\int_0^6 \int_0^y x \, dx \, dy.$$

Change the integral into an equivalent polar integral. **Do not evaluate**.

2. Using **Cylindrical** coordinates **set up**, **but do not evaluate**, the triple integral

$$\iiint_E xyz \, dV,$$

where E is the circular cylinder whose base is the circle $x^2 + (y-1)^2 = 1$ and whose top lies in the plane $z = 5 - x$.

3. Consider the triple integral $\displaystyle\iiint_D x \, dV$, where D is the wedge cut from the cylinder $x^2 + y^2 = 1$ by the planes $z = -y$ and $z = 0$ in the half-space $z \geq 0$.

   (a) Sketch the projection of D onto the plane you use to set up the triple integral. Label the sketch appropriately.

   (b) **Set up**, **do not evaluate**, this integral in **Cartesian (Rectangular)** coordinates.

   (c) **Set up**, **do not evaluate**, the triple integral in **Cylindrical** coordinates.

   (d) Evaluate the integral you set up in (c).

4. Consider the solid E in the first octant bounded between the planes $x + y + 2z = 2$ and $2x + 2y + z = 4$ with density $\delta(x, y, z) = xyz$.

   (a) Sketch E and label appropriately.

   (b) **Set up**, **do not evaluate**, a triple integral that computes the **volume** of E. Be sure to include a sketch of the projection.

   (c) **Set up**, **do not evaluate**, a triple integral that computes the **mass** of the solid E.

5. Set up, do not evaluate, the volume of the smaller region cut from the solid sphere $\rho \leq 2$ by the plane $z = \sqrt{3}$ in the first octant in **Spherical** coordinates.

6. Use the change of variable equations $x = 3r\cos\theta$, $y = 2r\sin\theta$ to find the volume of the region bounded by the xy-plane, the paraboloid $z = x^2 + y^2$, and the elliptical cylinder $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$. The half-angle formulas are useful:

$$\cos^2\theta = \frac{1 + \cos 2\theta}{2} \quad \text{and} \quad \sin^2\theta = \frac{1 - \cos 2\theta}{2}.$$

7. Let $C = C_1 \cup C_2 \cup C_3$, where $C_1 : y = x^2$ from $(0,0)$ to $(2,4)$; C_2 : line segment joining $(2,4)$ and $(2,0)$; C_3 : line segment joining $(2,0)$ and $(0,0)$. For the velocity field $\mathbf{F} = x\mathbf{i} - y\mathbf{j}$, find

   (a) The flow of $\mathbf{F}$ along C_1.

   (b) The outward flux of $\mathbf{F}$ across the closed path C if the outward flux of $\mathbf{F}$ over C_2 is -8 and the outward flux of $\mathbf{F}$ over C_3 is 0.

8. Consider the vector field $\mathbf{F}(x,y,z) = \langle 2xz, \sin z, x^2 + y\cos z \rangle$.

   (a) Show that $\mathbf{F}$ is conservative.

   (b) Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is the line path that connects $A(1,2,\pi)$ to $B(e,4,0)$, B to $C(1,2,-3)$, and C to $D(2,1,2\pi)$.