M261 EXAM III SPRING 2007

NAME:_____

SECTION NUMBER: _____

Problem	Points	Score
1	15	
2	15	
3	15	
4	15	
5	20	
6	20	
Total	100	

You may NOT use calculators or any references. Show work to receive full credit.

GOOD LUCK !!!

1. *The Cobb-Douglas Production Function:* By investing x units of labor and y units of capital, a low-end watch manufacturer can produce $P(x, y) = 50x^{0.4}\, y^{0.6}$ watches. Using Lagrange multipliers, find the maximum number of watches that can be produced on a budget of 20,000 dollars if labor costs 100 dollars per unit and capital costs 200 dollars per unit.

2. Given $f(x, y) = (x^2 + y^2)\, e^{-x}$ complete the following.

   (a) Find the critical points. There are two.

   (b) Compute the second-order partials and the discriminant.

   (c) Using the second derivative test classify the critical points.

3. Let $f(x, y, z) = \cos(x^2) + y^2 + e^z$

   (a) Find an equation for the plane tangent to the level surface $f(x, y, z) = 4$ at the point $(\sqrt{\pi}, 2, 0)$.

   (b) Define the linearization $L(x, y, z)$ at $(\sqrt{\pi}, 2, 0)$.

   (c) Find the parametric equation for the line normal to the level surface at $(\sqrt{\pi}, 2, 0)$.

4. Given that $\delta(x, y) = xe^y$ complete the following

   (a) Sketch the domain of integration for the integral given below and then change the order of integration. The graph should show a horizontal line segment with the endpoints defined in terms of y and a vertical line segment with the endpoints defined in terms of x. DO NOT EVALUATE the integral .

$$M = \int_1^9 \int_{\sqrt{y}}^3 \delta(x, y)\, dx\, dy$$

   (b) Using the results from part(a), define the integral M_x. DO NOT EVALUATE M_x. Does M_x/M give $\bar{x}$ or $\bar{y}$?

   (c) Define the integral for I_x. DO NOT EVALUATE.

5. Let $\mathcal{W}$ be the region above the unit sphere $x^2 + y^2 + z^2 = 6$ and below the paraboloid $z = 4 - x^2 - y^2$.

   (a) Using cylindrical coordinates show that the surfaces intersect when $r = \sqrt{2}$.

   (b) Compute the volume of $\mathcal{W}$.

6. Let D be the region bounded by $z = 4 - y^2$, $y = 2x$, $z = 0$, and $x = 0$. Draw the region. Write the integrals as equivalent iterated integrals in the order given. For each of these, include a two-dimensional graph of the projection of D onto the appropriate two-dimensional plane. DO NOT EVALUATE THE INTEGRALS.

   (a) $dV = dz\, dx\, dy$

   (b) $dV = dx\, dy\, dz$

   (c) $dV = dy\, dx\, dz$