MATH261 EXAM III FALL 2007

NAME:____________________________
SECTION NUMBER: ______

You may NOT use calculators or any references. Show work to receive full credit.

GOOD LUCK !!!

Problem	Points	Score
1	10	
2	10	
3	10	
4	14	
5	14	
6	14	
7	14	
8	14	
Total	100	

1. (a) Sketch the region in the xy-plane that is bounded by the graphs of $y = 5 - x^2$ and $y = 9 - 2x^2$. Within the region, include either a vertical or horizonta line segment with the gridpoints defined in terms of one variable. Find the points of intersection.

   (b) Set up and evaluate the integral that represents the mass of this region if the density is equal to the distance from the y-axis.

2. Consider $\int_0^{\sqrt{\pi/2}} \int_x^{\sqrt{\pi/2}} \int_1^3 \sin(y^2)\, dz\, dy\, dx$.

   (a) Graph the volume associated with the limits of integration. Assuming that $dV = dz\, dx\, dy$, include a line segment with the endpoints defined in terms of two variables. Next, project the 3D graph onto the appropriate two dimensions. On the 2D graph, include a line segment with the endpoints defined with one variable.

   (b) Evaluate the integral for $dV = dz\, dx\, dy$.

3. (a) Consider the volume bounded by the paraboloid $x^2 + y^2 + z = 6$, the cone $z = \sqrt{x^2 + y^2}$, and inside the cylinder $x^2 + y^2 = 1$. Graph the volume.

   (b) Assuming the volume has constant density $\delta(x, y, z) = \kappa$, find the mass, $\bar{x}, \bar{y}$ and $\bar{z}$.

4. Consider the region R bounded on the outside by the circle $r = 1$ and on the inside by the cardiod $r = 1 + \cos\theta$. Graph $r = 1 + \cos\theta$ in the $r\theta$-plane. Next, graph $r = 1 + \cos\theta$ and $r = 1$ in the xy-plane. Find the area of R.

5. Evaluate the following integral where D is the region bounded by the semicircle $x = \sqrt{a^2 - y^2}$. Include a sketch of D.

$$\int\int_D e^{-x^2 - y^2}\, dA =$$

6. Consider the volume E enclosed by $x^2 + y^2 + z^2 = 4$ lying above the plane $z = 1$. Set up but DO NOT evauluate the triple integral that represents the volume in spherical coordinates. For order, assume $d\rho\, d\phi\, d\theta$. Include a sketch of the region.

7. Using the Jacobian, find the area of the ellipse $\dfrac{x^2}{16} + \dfrac{y^2}{9} = 1$.

8. Integrate $f(x, y, z) = -\sqrt{x^2 + z^2}$ over the circle $\mathbf{r}(t) = (a\,\cos t)\mathbf{j} + (a \sin t)\mathbf{k}$ where $0 \leq t \leq 2\pi$.