

MATH261 EXAM II Spring 2008

NAME: _____

SI: _____

SECTION NUMBER: _____

You may NOT use calculators or any references. Show work to receive full credit.

GOOD LUCK !!!

| Problem | Points | Score |
|---------|--------|-------|
| 1 | 10 | |
| 2 | 10 | |
| 3 | 10 | |
| 4 | 10 | |
| 5 | 15 | |
| 6 | 15 | |
| 7 | 15 | |
| 8 | 15 | |
| Total | 100 | |

1. (a) Use the Chain Rule to find $\frac{\partial z}{\partial u}$ when

$$z = \tan^{-1}(x/y), \quad x = u \cos v, \quad y = u \sin v$$

- (b) Evaluate $\frac{\partial z}{\partial u}$ if $(u, v) = (1.3, \pi/6)$.

2. Find $\partial z / \partial x$ at the coordinate point $(-3, -1, 1)$ if the equation

$$xz + y \ln z - z^2 + 4 = 0$$

defines z as a function of the two independent variables x and y .

3. Given $f(x, y, z) = \cos(xy) + e^{yz} + \ln(xz)$

- (a) Find the gradient of f .

(b) Find the directional derivative of f at the point $P(1, 0, 1/2)$ in the direction of the vector $\langle 1, 2, 2 \rangle$.

(c) Give a direction in which the directional derivative at P is zero.

4. (a) Find an equation of the tangent plane to the level surface defined by

$$f(x, y, z) = \cos \pi x - x^2 y + e^{xz} + yz = 4$$

at the point $P(0, 1, 2)$.

(b) Find the equation of the normal line to the level surface at P .

5. (a) Find the critical point(s) for the function $f(x, y) = x^2 + 2xy$.

(b) The function $f(x, y) = x^3 - y^3 - 2xy + 6$ has critical points at $(0, 0)$ and $(-\frac{2}{3}, \frac{2}{3})$. For each critical point, determine if the function has a local minimum, local maximum, or saddle point. The answer should include the function value.

6. The temperature at a point (x, y) on a metal plate is $T(x, y) = 2x^2 - 8xy + 2y^2$. A bug on the plate walks around the circle of radius 5 centered at the origin. What are the highest and lowest temperatures encountered by the bug?

7. Sketch the area in the xy -plane that is bounded by the graphs of $f(x) = \frac{1}{2}x^2 - 2$ and $g(x) = x - \frac{1}{2}$. Find the mass of this area with respect to the density function $\delta(x, y) = x - y$.

8. Evaluate

$$\int_0^1 \int_{\sqrt{x}}^1 \sqrt{3 + 2y^3} \, dy dx.$$