M261 EXAM II SPRING 2007

NAME:_____

SECTION NUMBER: _____

You may NOT use calculators or any references. Show work to receive full credit.

GOOD LUCK !!!

Problem	Points	Score
1	15	
2	20	
3	20	
4	15	
5	15	
6	15	
Total	100	

1. Given

$$w = xy + yz + xz, \quad x = \cos u + \sin v, \quad y = \cos u \sin v, \quad z = uv,$$

complete the following:

   a. Find $\partial w/\partial x$, $\partial w/\partial y$, $\partial w/\partial z$

   b. Use the multivariable chain rule to express $\partial w/\partial v$ first in terms of x, y, z, u, v and then in terms of only u and v. DO NOT SIMPLIFY YOUR ANSWER. Include the appropriate tree diagram.

2. Find the limit, if it exists, or show that the limit does not exist.

   a. $\displaystyle\lim_{(x,y)\to(1,2)} \frac{x+3}{x} =$

   b. $\displaystyle\lim_{(x,y)\to(-2,-1)} \frac{x^2+6x+9+2xy+y^2+6y}{x+3+y} =$

   c. $\displaystyle\lim_{(x,y)\to(6,3)} xy\cos(x-2y)$

   d. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{2x^4+y^2}{x^2y}$

3. Consider the vector functions at π defined by

$$\mathbf{r}(\pi) = \left\langle 1, 0, \pi^2 \right\rangle, \qquad \mathbf{v}(\pi) = \langle 0, 1, 2\pi \rangle, \qquad \mathbf{a}(\pi) = \langle -1, 0, 2 \rangle.$$

   (a) Find the unit tangent vector, $\mathbf{T}(\pi)$.

   (b) Find the tangential component of acceleration, $a_{\mathbf{T}}$, at π.

   (c) Find the normal component of acceleration, $a_{\mathbf{N}}$, at π. DO NOT SIMPLIFY.

   (d) Find the unit normal vector, $\mathbf{N}(\pi)$. DO NOT SIMPLIFY.

   (e) Find the curvature of the curve at π. (Find κ.)

4. Given the function

$$f(r, \theta, z) = \frac{2z}{r^2 + \theta}$$

   (a) Find the first partial derivatives with respect to each variable. Clearly mark your answers with the name of the derivative (for example, $\partial f/\partial r$).

   (b) Find $f_{r\theta}$ and $f_{\theta z}$, clearly showing all work. Again, clearly mark your answers with the name of the derivative.

   (c) Find $f_{z\theta r}$.

5. Given the function

$$f(x, y) = \frac{1}{\sqrt{36 - x^2 - y^2}}$$

   a. Find the function's domain and range.

b. Find the boundary of the function with respect to the range.

c. Circle all items below that are true for the domain of this function:
open closed

bounded unbounded

6. The function $T(x, y, z) = (x^2 + y^2 + z^2)^{-1}$ describes the temperature at a point (x, y, z) in 3-space with some appropriate choice of units. Suppose you are at the point $P(1, 1, 0)$ in 3-space.

   a. What is the gradient vector of T at the point P.

   b. What is the direction of greatest increase in temperature from the point P?

   c. Find the directional derivative of T at the point P in the direction of $\langle 1, 0, -2 \rangle$.

   d. If you start at P in the direction of $\langle 1, 0, -2 \rangle$, will you be getting hotter or colder? At what rate?