Power series & Taylor series: Review exercises

1. Let $f(x) = \begin{cases} e^x + e^{-1/x^2} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$ and assume that $f^{(n)}(0) = 1$ for $n = 0, 1, 2, 3, \ldots$.

(a) What is the Maclaurin series for f?

(b) What is the interval of convergence of the Maclaurin series?

(c) For what values of x does $f(x)$ equal the sum of the Maclaurin series?

2. (a) Find the Maclaurin series for the function $f(x) = x\cos^2 x$. Write the result in closed form—as a sum $\sum_{n=0}^{\infty} a_n x^n$. (Hint: $\cos^2 x = \frac{1}{2}(1 + \cos(2x))$.)

(b) Find the sum of the series $\displaystyle\sum_{n=0}^{\infty} \left(\frac{x^2+1}{3}\right)^n$ as a function of x. What is the interval of convergence of the series.

3. (a) Find the first 5 terms of the Maclaurin series of the function $f(x) = (1 - 3x^2)^{-1/3}$.

(b) If the first 6 terms of the Maclaurin series of the function $f(x) = \dfrac{1}{\sqrt{1-x^2}}$ are

$$(1 - x^2)^{-1/2} = 1 + \frac{1}{2}x^2 + \frac{3}{8}x^4 + \frac{5}{16}x^6 + \frac{35}{128}x^8 + \frac{63}{256}x^{10} + \cdots,$$

find the first 7 terms of the Maclaurin series of the function $f(x) = \arcsin(x)$.

4. The Maclaurin series for $f(x) = \cos x$, $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$, converges for all x, $-\infty < x < \infty$. Show that the series converges to $f(x) = \cos x$ for all x.

5 (a) Find the Taylor series expansion for $f(x) = \ln(x)$ at $a = 4$. Write the result using summation notation.

(b) Find the first four terms of the Taylor series expansion for $f(x) = x^2 \ln(x)$ at $a = 4$.

6. For what values of x does the power series $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^{2n-1}}{2n-1}$ converge.

7. Find the Taylor series of $f(x) = x^3 - 2x + 4$ at $a = 2$.