Review Exercises : Series

1. Determine whether the series is convergent or divergent. In either case explain why. **You must justify your answers.**

(a) $\sum_{n=1}^{\infty} \frac{2^n}{(2n+1)!}$

(b) $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^3+n+1}}$

(c) $\sum_{n=1}^{\infty} ne^{-n^2}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{n3^n}$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^2+n+1}}$

2. Determine whether the series is absolutely convergent, conditionally convergent or divergent. In any case, **explain why.**

(a) $\displaystyle\sum_{n=3}^{\infty} \frac{(-1)^{n+1}}{n+2^n}$

(b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n 2^{1/n}$

3. Determine the radius of convergence and the interval of convergence (including the endpoints) of the series $\sum_{n=1}^{\infty} \frac{(x+4)^n}{n3^n}$.

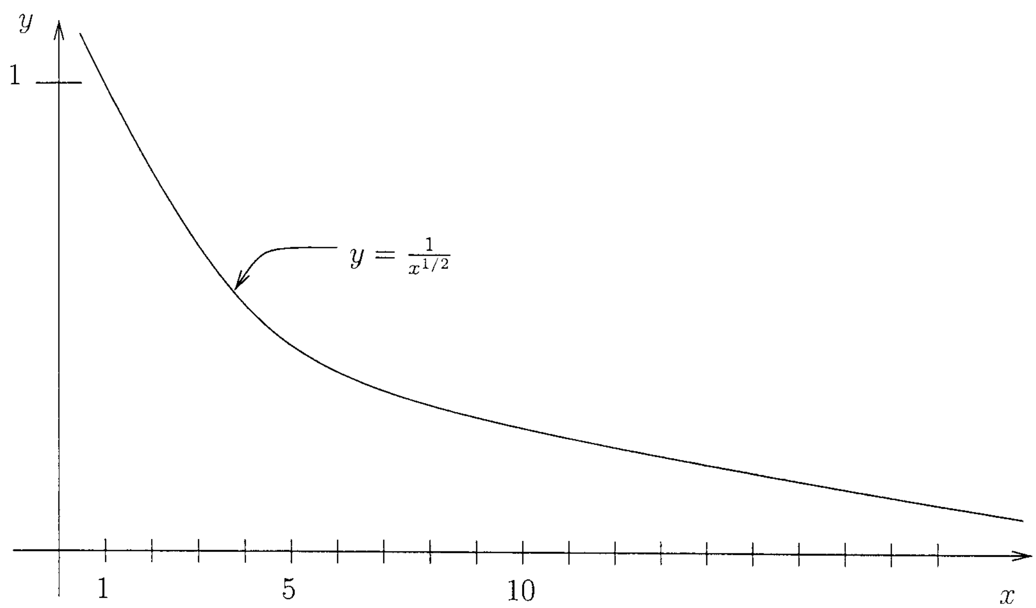
4. (a) Derive a formula (a closed form formula, not one that is the sum of n terms) for the Nth partial sums, s_N, of the series $\sum_{n=3}^{\infty} 2(0.4)^{2+n}$.

(b) If the sum of this series exists (i.e. if the series converges), then find the sum of the series using the formula for s_N (find an exact answer—no calculator). If the series does not coverge, then use the formula for s_N to explain why.

5 Consider the series $\sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$ and the function $f(x) = \frac{1}{\sqrt{x}}$, which is graphed below. Illustrate the proof of the integral test by drawing appropriate rectangles on the graph to deduce that

$$\sum_{i=1}^{n} a_i = \frac{1}{1^{1/2}} + \frac{1}{2^{1/2}} + \frac{1}{3^{1/2}} + \cdots + \frac{1}{n^{1/2}} \geq \int_1^{n+1} \frac{1}{x^{1/2}} dx.$$

Explain in your own words how this shows that the series $\sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$ is divergent.



6. Assume that the function y can be written as a power series

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + \cdots$$

Use this series to find a solution to the initial value problem (differential equation)

$$y - y' = 3 \text{ with } y(0) = 4.$$

Write your solution for y as a power series and then determine the function that is represented by the series (you should be able to see which function it is by the form of the power series).