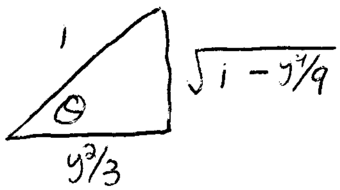
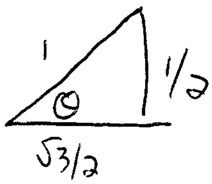
1a) $\theta = \cos^{-1}\left(\frac{y^2}{3}\right)$



$\sin\left(\cos^{-1}\left(\frac{y^2}{3}\right)\right) = \sqrt{1 - y^4/9}$

$$\boxed{\sqrt{1 - y^4/9}}$$

1b) $\tan\left(\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)$



$$\boxed{1/\sqrt{3}}$$

1c) $\cosh(5x) + \sinh(5x) = e^{5x}$

$$\boxed{e^{5x}}$$

2a) $\frac{d}{dx}\left(3\sinh(x^2)\right) = 3$

$$\boxed{3\cosh(x^2) \cdot 2x}$$

2b) $\frac{d}{dx}\left(x^{\sqrt{x}}\right)$

$$y = x^{\sqrt{x}}$$

$$\ln y = \sqrt{x}\,\ln(x)$$

$$\frac{1}{y}\frac{dy}{dx} = \sqrt{x} \cdot \frac{1}{x} + \ln(x)\frac{1}{2}x^{-1/2}$$

$$\frac{dy}{dx} = y\left(\frac{1 + (1/2)\ln(x)}{\sqrt{x}}\right)$$

$$\boxed{\frac{dy}{dx} = x^{\sqrt{x}}\left(\frac{1 + (1/2)\ln(x)}{\sqrt{x}}\right)}$$

2c) $\frac{d}{dx}\left(\sin^{-1}(x^2)\right) =$

$$\boxed{\frac{2x}{\sqrt{1 - x^4}}}$$

2d) $\frac{d}{dx}\ln\left(\frac{3}{x}\right) = \frac{1}{3/x}\left(-\frac{3}{x^2}\right) = \boxed{-\frac{1}{x}}$

<u>or</u> $\frac{d}{dx}\left(\ln(3) - \ln(x)\right)$

3a) $\displaystyle\int \frac{e^x}{e^{2x}+1}\,dx$ $\qquad u = e^x \quad du = e^x dx$ $\qquad \boxed{\tan^{-1}(e^x) + C}$

$$= \int \frac{du}{1+u^2} = \tan^{-1}(u) + C$$

3b) $\displaystyle\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$ $\qquad u = \sqrt{x}$

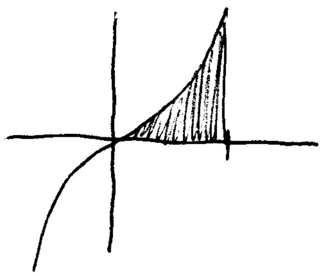
$du = \frac{1}{2}\frac{1}{\sqrt{x}}$

$$= 2\int_{x=1}^{4} e^u\,du = 2e^u\Big|_{x=1}^{4} = 2e^{\sqrt{x}}\Big|_1^4 = \boxed{2e^2 - 2e}$$

3c) $\displaystyle\int \sec^2(x)\, e^{\tan x}\,dx = \int e^u\,du = e^u = \boxed{e^{\tan(x)} + C}$

$u = \tan(x) \quad du = \sec^2 x\,dx$

☆ 4

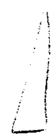


$$\int_0^{\pi/3} \tan x\,dx = \int \frac{\sin x}{\cos x}\,dx$$

$u = \cos x$

$$= -\int \frac{1}{u}\,du = -\ln|u|$$

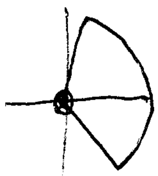
$$= -\ln|\cos x|\Big|_0^{\pi/3}$$ $\qquad \boxed{\ln 2}$



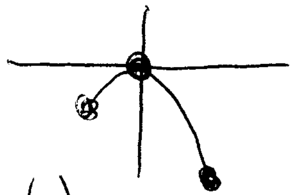
$$= -\ln(\cos(\pi/3)) + \ln(\cos(0))$$

$$= -\ln(1/2) + \ln(1) = \ln(2)$$

☆ 5) $y = \ln(\cos x)$ absolute max/min $[-\pi/4, \pi/3]$



$$\frac{dy}{dx} = \frac{1}{\cos x}(-\sin x)$$



$\boxed{\text{max } 0}$

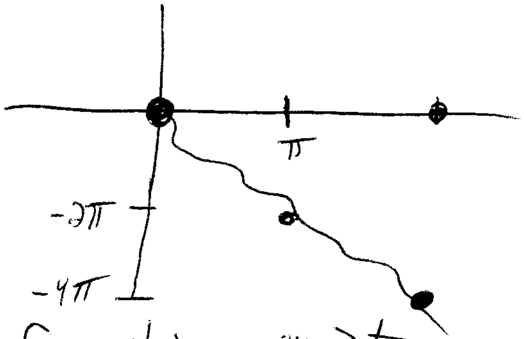
$\boxed{\text{min } -\ln(2)}$

$\frac{dy}{dx} = 0$ when $x = 0$ then $y = \ln(1) = 0$

$x = -\pi/4 \quad \cos x = \sqrt{2}/2 \quad \ln(\sqrt{2}/2) = -\ln(\sqrt{2})$

$x = \pi/3 \quad \cos x = 1/2 \quad \ln(1/2) = -\ln(2)$

6) $f(x) = -2x + \frac{1}{2}\sin(2x)$



$f'(x) = -2 + \cos(2x)$

$f'(x) < 0$ everywhere

decreasing on $[0, 2\pi]$

a) so it's 1-to-1, so inverse function exists

b)

domain f $[0, 2\pi]$ range f $\cancel{[0, -4\pi]}$ $[-4\pi, 0]$

range f^{-1} $[0, 2\pi]$ domain f^{-1} $\cancel{[0, -4\pi]}$ $[-4\pi, 0]$

c) $y = -2x$

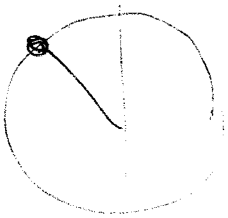
find $f^{-1}(x)$?

$\boxed{\text{impossible}}$

d) $\left.\frac{df^{-1}}{dx}(x)\right|_{x = -\frac{3\pi}{4} + \sqrt{2} = f\left(\frac{3\pi}{8}\right)}$

$$= \frac{1}{f'\left(f^{-1}\left(-\frac{3\pi}{4} + \sqrt{2}\right)\right)} = \frac{1}{f'\left(\frac{3\pi}{8}\right)} = \frac{1}{-2 + \cos\left(\frac{3\pi}{4}\right)}$$

$$= \frac{1}{-2 - \frac{\sqrt{2}}{2}} = \boxed{\frac{-2}{4 + \sqrt{2}}} = -.369$$



7) a) $\log_3(x^6)$ $\ln(2x)$

$\frac{d}{dx}\log_a(u) = \frac{1}{u\ln(a)}\frac{du}{dx}$

$= \frac{6}{\ln(3)}$

$\lim_{x\to\infty} \frac{\log_3(x^6)}{\ln(2x)} = \lim_{x\to\infty} \frac{\frac{1}{x^6\ln 3}\cdot 6x^5}{\frac{1}{2x}\cdot 2}$

$\boxed{\log_3(x^6) \text{ \& } \ln(2x) \text{ grow at same rate}}$

b) x^3+5x^2 & $4x^4$

$\lim_{x\to\infty} \frac{4x^4}{x^3+5x^2} = \lim_{x\to\infty} \frac{4}{\frac{1}{x}+\frac{5}{x^2}} = \frac{4}{0} = \infty$

$\boxed{4x^4 \text{ grows faster}}$

8) $\frac{dy}{dx} = 2y$ $y(0)=50$

find y

$\frac{dy}{y} = 2dx$

$\ln|y| = 2x + C$

$|y| = e^{2x}e^{c}$

$y = Ae^{2x}$

$\boxed{y = 50e^{2x}}$

9) half-life of radium is 1600 years

$m_0 = R$ when $t=0$

i) diff eq. $\boxed{\frac{dy}{dt} = -ky}$

ii) solve $y = Ae^{-kt} = \boxed{m_0e^{-kt} = y}$

iii) formula. $\boxed{y = m_0\left(\frac{1}{2}\right)^{t/1600}}$ $\boxed{y = m_0 e^{-\ln(2)t/1600}}$