Exam practice : mostly chapter 8 due 12/6 Tues

1. Calculate the following integrals. You must show your work. These integrals must be integrated analytically. If you just give the result from your calculator, you will get zero credit.

(a) $\int \underbrace{x^{1/3}}_{dv} \underbrace{\ln x}_{u}\, dx$

$$\int f'g = fg - \int fg'$$

$$= \frac{1}{1+\frac{1}{3}} \cdot x^{\frac{4}{3}} \ln|x| - \int \frac{3}{4} x^{\frac{4}{3}} \cdot \frac{1}{x}\, dx = \underline{\frac{3}{4} x^{\frac{4}{3}} \ln|x| - \left(\frac{3}{4}\right)^2 x^{\frac{4}{3}} + c.}$$

(b) $\int \frac{x}{x-1}\, dx = \int 1 + \frac{1}{x-1}\, dx = x + \ln|x-1| + c$

reducing to ~~the state~~ proper form.

(c) $\int \frac{1}{x^2-1}\, dx$

partial fraction.

$$\frac{1}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1} = \frac{A(x+1) + B(x-1)}{(x-1)(x+1)}$$

$$\therefore \quad 1 = 2A. \quad A = \frac{1}{2}.$$

$$1 = -2B \quad B = -\frac{1}{2}$$

$$\int \frac{\frac{1}{2}}{x-1} + \frac{-\frac{1}{2}}{x+1}\, dx = \frac{1}{2}\ln|x-1| - \frac{1}{2}\ln|x+1| + c$$

$$= \frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| + c$$

3. Calculate the following integrals. You must show your work. These integrals must be integrated analytically. If you just give the result from your calculator, you will get zero credit.

(a) $\int_1^2 x^4 \ln x \, dx$ Integration by part.

check basic examples

(b) $\int_{-3}^{-2} \frac{1}{1-t^2} \, dt$ partial fraction

$\frac{1}{(1-t)(1+t)}$ check.

(c) $\int_0^2 \frac{1}{\sqrt{x}} \, dx$ $\int x^{-\frac{1}{2}} dx$. basic integration formulas

(d) $\int \frac{1}{(1-x^2)^{3/2}} dx$ $\boxed{\text{trig } \cancel{\text{integ}} \text{ substitution}}$

basic form

$x = \sin t$

$dx = \cos t \, dt$

$\int \frac{1}{(1-\sin^2 t)^{3/2}} \cos t \, dt = \int \frac{\cos t}{\cos^3} dt = \int \frac{1}{\cos^2 t} dt$

3. Calculate the following integrals. You must show your work. These integrals must be integrated analytically. For definite integrals give exact answers— no calculator approximations. If you just give the result from your calculator, you will get zero credit.

(a) $\cancel{\int_2^3 x^{1/3} \ln x \, dx}$

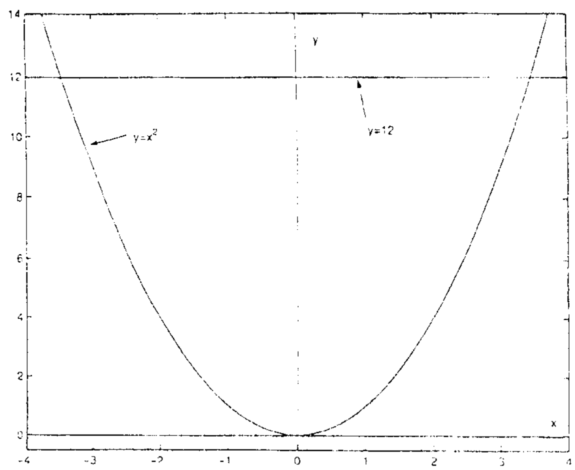
(b) $\int \frac{1}{\sqrt{1-x^2}} dx$; trig substitution

$\sin^{-1}(x) + C$

(c) $\int_2^3 \frac{x}{x^2-1} dx$ substitution.

(d) $\int \sin^3 x \cos^2 x \, dx$ $= \int \sin x \, (1-\underbrace{\cos^2 x}_{t}) \cos^2 x \, dx$

$= \int (t^2-1) t^2 \, dt$. [illegible]



4. Solve the following differential equation: $\frac{dy}{dx} = \frac{1+x}{xy}$, $x > 0$, $y(1) = -4$. (Solve for y).

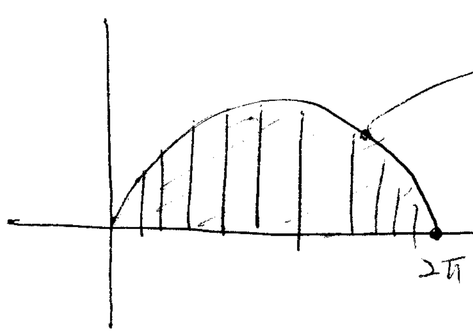
$$y \cdot dy = \frac{1+x}{x}\, dx$$

$$\frac{y^2}{2} = \int \frac{1+x}{x}\, dx + c = x + \ln|x| + c'$$

$$\boxed{y = -\sqrt{2x + 2\ln|x| + 14}} \Leftarrow \quad y = \pm\sqrt{2x + 2\ln|x| + c}$$

$$y(1) = -\sqrt{2 + c} = -4. \quad \therefore \boxed{c = 14}$$

5. Find the area under one arch of the cycloid $x = t - \sin t$, $y = 1 - \cos t$, $0 \le t \le 2\pi$.



$(x, y) = (t - \sin t,\ 1 - \cos t)$.

$$dA = y \cdot dx = y \cdot \frac{dx}{dt} \cdot dt.$$

$$\therefore A = \int_0^{2\pi} (1 - \cos t)(1 - \cos t)\, dt = \int_0^{2\pi} (1 - \cos t)^2\, dt$$