AN ALTERNATIVE CALCULUS I COURSE

Introduction

Mathematics is foundational to STEM disciplines and an in-depth understanding of mathematics as well as demonstrated proficiency is fundamental for success in STEM programs. Calculus I is often identified as a gateway course into STEM disciplines – serving as one of the first benchmarks for students. The student experience in such an introductory course can be a deciding factor of whether or not a student chooses to continue on a path toward a STEM degree. Unfortunately, Calculus I at post-secondary institutions has historically been perceived as a “filter” that blocks access to professional careers in STEM fields [15]. This perception is further validated when the failure rate in the course is high and experiences are consistently negative.

Failure in Calculus I is commonly identified in the literature as a grade of D or F or a withdrawal from the course (DFW). While a grade of D is not a failing grade in the eyes of a university, such a grade does not indicate that a student is adequately prepared for success in the courses subsequent to Calculus I. Findings from the MAA National Calculus Study have revealed that the DFW rate for Calculus I at research institutions is 25%, which is problematic for STEM majors [3].

Calculus I is a prerequisite for continuing on in a STEM program. Therefore, a student’s failure in this course can create barriers which negatively impact his or her decision to remain in the program of study. As Seymour and Hewitt [14] have documented, negative experiences in introductory courses are a contributing factor to students leaving STEM disciplines. This resonates with findings from the recent MAA National Calculus Study. In a survey to colleges and universities, Bressoud and Rasmussen [4] found that Calculus I, an introductory
STEM course, lowers confidence, decreases enjoyment of mathematics, and reduces the desire to continue in STEM fields. Poor instruction is often to blame for such experiences [4,14]. As a foundational STEM course, the importance of Calculus I in retaining STEM majors is evident and improving success rates is pertinent. The focus for this paper will be on a Calculus for Physical Scientists I course (referred to as Calculus I in this narrative) offered at our institution and discuss strategies for instruction in a Calculus I classroom.

**Calculus I at Our Institution**

Calculus I at our institution is comprised primarily of students majoring in Engineering, Mathematics, Physics, Computer Science, Statistics, and Chemistry. Total enrollment in Calculus I varies but usually ranges 375-450 students each semester. Graduate Teaching Assistants (GTAs) typically teach regular sized (30-40 students) sections and senior GTAs or experienced instructors teach the large sections (60-100 students).

The DFW rate in Calculus I at our institution is high. While the national failure rate at research universities is concerning, it is much lower than the current DFW rate at our institution, which has historically remained steady at about 40%. Though recent course redesign efforts undertaken by a new course coordinator have started to result in a drop in the DFW rate, more is being done to address this concern. We will highlight one of the successful efforts undertaken – a two-semester version of Calculus I.

**An Alternative Version of Calculus I**

Traditional, non-engaging, and teacher-centered instruction is one of the contributors to low success rates in Calculus I, and unfortunately the way that mathematics courses are often taught [2]. Such instruction typically results in shallow, unengaged learning where students are watching mathematics being done rather than actually doing mathematics. Teacher-centered instruction does not lend itself to students having a deep understanding of their own learning processes. Literature demonstrates that effective teaching and learning practices
involve much more than having students sit quietly in their desks while solutions to mathematics problems are perfectly constructed on the board.

However, in addition to the need for improved instruction, there are students at our institution who do not feel prepared to keep up with the pace of the one-semester version of Calculus I. This is often due to their weak algebra and trigonometry skills as well as poor study skills. Additionally, students often lack the confidence to persevere through the challenges that arise in Calculus I. Therefore we developed and piloted a two-semester version of Calculus I. Students self-selected to enroll into the course based on a recruitment email that discussed common problems that Calculus I students encounter at our institution and described an alternative course intended to address these obstacles. An excerpt from the recruitment email for the two-semester course is below:

Specifically, the course will involve an initial review in algebra and trigonometry, and will then explore topics of calculus at a slower rate than a semester long course would do. Additionally, time will be spent honing study skills and doing mathematics in small groups. My goal will be to establish an active, student-centered learning environment in which we develop, discuss, and deepen our understanding of topics in Calculus.

While the recruitment email was sent to all students registered for Calculus I and students self-selected into the course, this email was targeted at students who felt they were underprepared for the course and wanted a stretched out version. There is a population of students that arrive at our institution who are seeking to major in an Engineering program, but have yet to be accepted. Their major is listed as Undeclared with an Engineering Intent. Only about 15% of these students end up getting accepted and completing such a degree and many have not had a calculus course prior to entering. Most of the students in the two-semester course were pursuing a STEM major, with many of them seeking, but not yet accepted into an Engineering program.

In addition, nearly all of the students enrolled in the two-semester course had not taken a calculus course beforehand. This is worth noting as it is not uncommon for a Calculus I
course to consist of large number of students who have had calculus before (about 60-70% of students enrolled), and this often has a negative impact on the students who have not previously taken calculus [5]. The two-semester Calculus I course was intended to provide a more accessible path for such students and provide one mechanism through which students who are excited about working in a STEM field would persist in a STEM program. The alternate version of the course met three days a week over two semesters, had a three-week review of prerequisite knowledge at the beginning of the course, and incorporated pedagogy that was chiefly student-centered.

**Pilot Implementation**

During the 2013-2014 academic year we piloted a two-semester Calculus I course whose framework for instruction included problem-based learning (PBL), group learning, and regular mathematical dialogue. The purpose of the course was to (1) establish an active, student-centered learning environment in which students could develop, discuss, and deepen their understanding of topics in Calculus I, (2) foster the development of learning skills that transcend Calculus I, and (3) encourage the development of a community of Calculus I learners.

**Framework for Instruction**

The two-semester Calculus I course was designed to establish a strong foundation in Calculus I content and to give students the skills needed to succeed in subsequent mathematics courses. Curriculum and instruction evolved and adapted to meet students’ needs. An active, student-centered learning environment was established in which students could deepen their understanding of topics in Calculus I as they developed into a community of learners. The course highlighted evidence-based practices and gave students the skills to regulate their own learning. However, it is not enough to simply put students in groups or create good problems where students can still remain passive observers. Good teaching
practices were emphasized, such as clearly articulated expectations, connecting content to real-life, willingness to meet and have conversations with students, responding to student questions, creating an environment of mutual respect, etc. These practices combined with evidence-based, active pedagogy created a classroom culture conducive to effective teaching and learning.

**What Class Looked Like**

Group activities occurred daily in the course, with all group members expected to contribute to activities and discussion. These practices and expectations were established at the beginning of the first semester and maintained throughout the duration of the course. The intent was for students to participate in a positive, collaborative setting in which knowledge could be developed and shared. This set the tone for the course. Students recognized that collaboration and learning through mistakes was part of the culture of the course, stating, “The whole attitude and mindset of the class was very welcoming [to] struggling through learning calculus” and the classroom “was a comfortable environment and conducive to learning.” In general the interviews reflected that students found working in groups to be invaluable as it provided the opportunity to see a variety of perspectives in working on problems. Norms of the class were clear and an environment was immediately established so that these expectations and behaviors would be maintained for the duration of the course.

Students often engaged in problem-based learning (PBL) while working in small groups. PBL is well-established as an instructional strategy that has a positive impact on learning [6, 8, 9] and on motivation to learn [7]. One such model of PBL, Treisman's Emerging Scholars Program (ESP), is designed as a workshop in which students work together in small, collaborative learning groups on challenging mathematics problems on a regular basis throughout the semester and has been demonstrated to improve calculus students’ learning outcomes [10].
Motivated by the ESP model, we designed in-class problems and activities that drew on multiple constructs from course content and guided students to build new knowledge. PBL was used as a mechanism to encourage engagement in learning and encourage collaboration \cite{1, 10}. Two facilitators (the instructor and a course assistant) were present while students worked on problems. Facilitators provided guiding prompts, such as “how” and “why” questions, requiring students to support their ideas. This encouraged small group and whole-class discussion. Students valued the discussions that were motivated by carefully crafted problems, commenting

“If I didn’t know how to solve a math problem…somebody would speak up and try to attack the problem.”

“The real world examples – things that you don’t think that you use math for…definitely changed the way I think about mathematics.”

Collaboration on application problems helped students develop confidence in their ability to explore difficult math problems and develop new ways in thinking about mathematics.

In addition to the PBL activities, daily reflections were also an important aspect in the design of the course as they required students to reflect back on the class period and their understanding of the material that was covered. These were intended to encourage students to be more mindful of the content they understood well and the content with which they struggled.

Sample class activities

As part of the initial review, students needed to get reacquainted with functions, domain, and range. Specifically, students struggled with modeling functions for real-life situations. To address this issue, students began the class with an activity that emphasized these concepts. An excerpt of the activity is included below.
Overall, we wanted students to be able to engage in challenging problems and have meaningful discussion to deepen their understanding of the content. Most class periods began with an opening problem with reflection. The opening problem was written to be on the cusp of new knowledge – a challenging, yet accessible problem that incorporated recently learned ideas and fostered the development of new knowledge. For example, following a class period in which left, right, and midpoint methods were introduced for calculating area under a curve, students were asked about which method gives the “best” area approximation.

Activity 1: Modeling - Groups, No Calculator Allowed

Styrofoam cups have a bottom part and a lip part:

What do you notice when several cups are stacked together?

For 2 cups, there are _________ bottom parts and _________ lip parts.
For 4 cups, there are _________ bottom parts and _________ lip parts.
For 8 cups, there are _________ bottom parts and _________ lip parts.

Create a model for the total height \( h \) of a stack of \( c \) cups.

What are the domain and range?

Are there any values of \( c \) for which the model does not work or make sense? (Class Discussion).

Overall, we wanted students to be able to engage in challenging problems and have meaningful discussion to deepen their understanding of the content. Most class periods began with an opening problem with reflection. The opening problem was written to be on the cusp of new knowledge – a challenging, yet accessible problem that incorporated recently learned ideas and fostered the development of new knowledge. For example, following a class period in which left, right, and midpoint methods were introduced for calculating area under a curve, students were asked about which method gives the “best” area approximation.
Warm-Up Problem & Reflection

Warm-Up Problem:

You want to estimate the area underneath the graph of a positive function by using four rectangles of equal width. The rectangles that will give the best estimate of this area are those with height obtained from the:
(a) Left endpoints
(b) Midpoints
(c) Right endpoints
(d) Not enough information

Discuss.

At the end of the class period, students filled out a reflection, which was on the backside of the paper with the warm-up problem:

Daily Reflection (to be filled out before class ends)

On a scale from 0 to 100%, rate how well you understand today’s class, where 100% = understood everything, 50% = understood half of the class, and 0 = understood nothing.

What math content questions do you have? (What was unclear? How does the day’s lesson relate to other math concepts?) If you cannot think of a question, write a possible test question.

Tell me something else you think I should know.
These daily reflections provided a private, safe setting for students to ask questions without feeling embarrassed about their questions, especially at the beginning of the course when students had yet to adjust to the culture of the classroom. Giving students sufficient time to respond on the reflections was essential for getting meaningful replies and fostering student self-reflection.

Through the development of the course the reflections have changed somewhat. Providing more structure for students has increased the meaningful responses and self-reflection. Below is an updated reflection that is currently used in the course.

<table>
<thead>
<tr>
<th>Daily Reflection (to be filled out before class ends)</th>
</tr>
</thead>
<tbody>
<tr>
<td>What was the muddiest part of today’s class and how do you feel about the content? (place a mark on the scale below)</td>
</tr>
<tr>
<td>Stuck in the mud ———— Squeaky clean</td>
</tr>
<tr>
<td>What could you use more practice on?</td>
</tr>
</tbody>
</table>

So far, in Chapter 2, we have talked about limits, continuity, and limits involving infinity. You have an exam in just over a week, have you begun studying/reviewing these topics? If so, how are you spending your time studying?

Tell me something else you think I should know.

Regardless of format, daily reflections can provide a mechanism for developing metacognitive abilities specifically connected to self-regulated learning. Metacognition is an important piece of problem solving and can positively influence academic achievement [12, 13]. For these reasons, we hope to further explore the impact these reflections can have on self-regulated learning through a research study.
Impact on Students

We collected quantitative data on the students enrolled in the pilot two-semester Calculus I course and conducted interviews with six students from the course. The quantitative data collected included all Calculus I course work grades and any final grades for Calculus II and III. Interviews were conducted with students one year after completing the course.

Students (N=279) in the spring 2014 one-semester Calculus I course took the same final exam as the students (N=13) in the two-semester Calculus I course. Students in the two-semester course performed significantly better than those in the one-semester course ($t(13.902) = 3.5955, p = 0.003$). However, with such vastly different sample sizes, it is more meaningful to discuss the effect size. Cohen’s $d$ was 0.8, indicating a large effect and that the course made a positive impact on students [11].

Of the N=13 students in the two-semester course, eight continued on to Calculus II (the remaining five students did not need the course for their majors). Seven of the eight students received a grade of A or B in Calculus II; one of the eight students received a grade of C in Calculus II. At this point all of the eight students have either taken or are currently taking Calculus III. Of the students who have taken Calculus III, all have received a grade of A, B, or C in the course.

Five of the N=13 students entered our institution as Undeclared with an Engineering Intent. All five have now been accepted into the Mechanical Engineering program, which is of note since such a small percentage of Engineering seeking students end up getting admitted into an Engineering program. The two-semester course has had lasting effects on student attitudes as well. We see this not only in student grades beyond Calculus I, but also in their interview comments:

“The knowledge I learned there was solidifying forever.”

“I feel like we came out at the end at the final better off than the [one-semester calculus] students.”
"Yes [the course] made me feel like I could make it to the end; it really did make me feel all is possible."

Quantitative and qualitative data from this pilot study not only show an impact on students’ academic performance, but are also informing a possible more developed research study in which specific components of the course, such as group work and self-reflections, can be studied more deeply.

**Impact at our Institution**

This alternate course has since become a permanent course at our institution with enrollment numbers increasing each year. Our recruitment statement goes through continual changes and students commonly ask whether they should take the one-semester or two-semester Calculus I course. To help students navigate this decision, we provide the following statement to students:

- Upon entering Calculus I, you should have a working understanding of mathematical vocabulary words such as (but not limited to): function, domain, range, linear, polynomial, piecewise, expression, equation.
- Can you do most of the following types of problems without the use of a calculator or references? If you cannot, then you should consider taking the two-semester version of Calculus I.
  - Determine the domain and range of a function.
  - Evaluate functions and understand function notation.
  - Add, subtract, multiply, and divide fractions.
  - Factor polynomials.
  - Algebra with exponents.
  - Evaluate the 6 trigonometric functions at the angles: 0, \( \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2} \).
  - Simplify rational expressions.
  - Graph the following functions:
    \[
    \begin{align*}
    y &= mx + b \\
    y &= ax^2 + bx + c \\
    y &= x^3 \\
    y &= |x| \\
    y &= \sqrt{x}
    \end{align*}
    \[
    \begin{align*}
    y &= \sin(x) \\
    y &= \cos(x) \\
    y &= \tan(x)
    \end{align*}
    \]

We also provide this statement to advisors so that they can assist students during summer registration. While this statement is only a portion of the recruitment email for the course, it is quite helpful when students are faced with the decision of which course to take.

As course enrollment increases and the class continues to be a success, we hope to determine whether or not this course will begin to (1) improve retention in STEM at our
institution and (2) provide an avenue to success for students who have not had calculus
before or students who are Undeclared with an Engineering Intent, as these students tend to
be in a group that are high risk for failing Calculus I at our institution. We also hope to start
studying the effects of the course on metacognition and self-regulated learning, as these were
unexpected behaviors that were observed of the students and warrant further investigation.

A common question that is asked of us with regard to this course is if we plan to offer
two-semester versions for Calculus II and III. Currently the answer is ‘no’. These students do
not need stretched out versions of subsequent courses. What they do need is time to develop a
strong foundation in Calculus I which gives them the tools they need to be successful in
Calculus II, Calculus III, and other relevant courses, regardless of factors such as
instructional format and course design.

Course materials are continually changing and adapting based on the needs of the
students currently enrolled in the course and the interactions we observed during this pilot
study. Writing, a tool known in the literature to be an effective component of instruction, is
also more purposefully incorporated into the materials as it allows students to practice
organizing their thoughts and ideas. In addition, results from this study are informing the
GTA training program at our institution – specifically for Calculus I, with future
implementations with Calculus II. We are sharing material and instructional practices through
training as well as the student comments made in interviews, as these provide unique insight
into Calculus I students.

Programmatic, instructional, and curricular changes are part of ongoing efforts to increase
student achievement, which we hope will address the many challenges that arise in the
retention of STEM majors. While there are many facets to this problem, we can partially
address this through the first-year mathematics experience. Thus, we aim to improve Calculus
I instruction, in particular, and provide alternative paths that enable students to be successful.
References


