Engaging Future Teachers in Problem-based Learning with the Park City Mathematics Institute Problems

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Engaging Future Teachers in Problem-based Learning with the Park City Mathematics Institute Problems

Mary E. Pilgrim

Abstract: Problem-based learning (PBL) is a pedagogical technique recommended for K-12 mathematics classrooms. However, the mathematics courses in future teachers’ degree programs are often lecture based. Students typically learn about problem-based learning in theory, but rarely get to experience it first-hand in their mathematics courses. The premise for trying an entirely problem-based approach to a capstone course for pre-service secondary mathematics teachers was simple: If the expectations of pre-service mathematics teachers is to engage their own students in the Common Core State Standards Mathematical Practices, then such classroom practice should be modeled in their undergraduate mathematics instruction. The Park City Mathematics Institute has developed several sets of problem-based materials around coherent mathematical themes, for use in 3-week intensive summer in-service workshops, which provide a great resource for pre-service mathematics courses. These materials were implemented with the single overarching goal in mind: To better equip pre-service teachers to use PBL by having them experience PBL for themselves. This article describes the results from both the students’ and the instructor’s perspectives and assesses how well the goal was achieved.

Keywords: Problem-based learning, pre-service mathematics teachers, mathematics education.

1. INTRODUCTION

During the Fall 2011 term I agreed to take on the daunting task of implementing a set of Problem-based Learning (PBL) materials in a capstone mathematics course for pre-service mathematics teachers. I use the word “daunting” because that was my initial reaction. Though I regularly incorporate group activities and technology explorations into all my classes, I tend to fall into the role of lecturer more often than I would like.

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When I reflect on my own undergraduate and graduate mathematics student experience, I realize that 100% of my mathematics courses were lecture based. Literature points out that teachers will often teach how they were taught [1, 3, 6]. As I now teach mathematics to pre-service mathematics teachers at the college level, I have become acutely aware of the need to model a variety of pedagogical styles, and I worry about the impact of a steady diet of lecture format in mathematics classes on future K-12 teachers.

K-12 teachers are often expected to implement instructional techniques that are more constructivist in nature, yet they themselves tend to have little to no experience in such a classroom. What outcomes can we expect from new teachers implementing unfamiliar instructional techniques in their classrooms? As Alesandrini and Larson note:

> Until teachers experience constructivism themselves, they may not be equipped to plan and facilitate constructivist activities [1].

It is for these reasons that I felt it important to give pre-service mathematics teachers an instructional experience other than lecture. Thus, I ventured into the unknown (for myself at least) - implementing PBL with pre-service mathematics teachers. Throughout this process I was focused on a single goal: To better equip pre-service teachers to use PBL by having them experience PBL for themselves.

2. FRAMEWORK

PBL has its roots in the medical field. In this student-centered instructional approach knowledge and skills are developed through research and the application of what is already known. PBL is an opportunity for students to construct knowledge based upon the problem presented and their current knowledge and skills. Having ill-structured problems that push students to raise questions is essential for effective PBL [4, 9, 10].

With this in mind, my classroom became constructivist in nature, with knowledge being built by multiple participants with varying perspectives [2]. It was through the constructivist lens that I approached my data collection and analysis as well. For me, qualitative data would play a key role in assessing the impact of PBL on my pre-service teachers.

3. DATA COLLECTION

The qualitative data collected were composed of; (i) student quotes recorded in a daily teaching journal by me; (ii) student writing from class journals; and (iii) the type of end-of-term presentations by the students. Student quotes and written comments were analyzed for common threads and patterns. End-of-term presentations were one of two types; lecture presentation or PBL
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Below is the first part of the first lesson from the 2008 Park City Mathematics Institute (PCMI) materials [8]:

**Important Stuff**
Come on, you can do it!

Picture a piece of graph paper. Now picture a dot at each intersection. We will call this *square dot paper*. A 5-by-5 piece of square dot paper would have five dots in each direction—also known as a “geoboard” (Figure 1). But the dot paper can be any size, really. We will say that the distance from a dot to its nearest neighbor is 1.

Line segments drawn on square dot paper must start and end at dots, but can be horizontal, vertical, or diagonal at any angle.

**PROBLEM**
On a 6-by-6 piece of square dot paper, what lengths of segments are possible?

Stuff in boxes is more important than other Important Stuff!

There is another type of dot paper, made from a grid of equilateral triangles instead of squares. This is usually called *isometric dot paper* (Figure 2). A 5-by-5 piece of isometric dot paper looks like a “squished” version of the square dot paper, but still has 25 dots. We will say that the distance from a dot to its nearest neighbor is still 1.
Following is a selection of student comments regarding the first lesson followed in parenthesis by initial data category:

When coming to this class, I was expecting it to be another lecture math class where we just took notes and then were left to our own devices to really understand the material. (Class Structure)

I thought the structure of the lesson was extremely helpful in achieving [the] goals. With the graph paper we were able to see the segments as a high school student might see them. (Class Structure)

I was a bit confused as to the importance of the boxes [and] numbered problems. . . maybe a bit more of an introduction as to what we’re supposed to focus on would have been good. (Class Structure and Course Content)

More explanation of the way to find the diagonal lengths of isometric dot paper would have helped. (Course Content)

I was very satisfied with the structure of the lesson. Our group collaborated well; we each approached the task differently at first, but were able to incorporate each member’s strategies and insights in some manner as we worked through the problems. (Class Structure)

When I began the lesson, I assumed that the lesson was intended to re-introduce us to geometric patterning and critical thinking in math. (Course Content)

I began to sift through the qualitative data coding based on the two categories: (i) Class Structure; and (ii) Course Content. However, as I coded data from the first session, it became clear that the classification of categories needed to be finer within each initial broad category. After a conversation with a colleague during which we read through de-identified student quotes, a new set of coding categories emerged, and I began identifying and coding data based on the following.

1. Class Structure: (i) group learning; and (ii) materials in PBL format
2. Content: (i) Goals of a lesson, and (ii) making connections in the content.

Through this process I was able to identify common threads and patterns in the data.

4. MODELING A PEDAGOGICAL PRACTICE

I had never utilized a set of classroom materials designed specifically for students to work through in groups as a replacement for lectures. In addition,
most of the students in the class had never experienced such a non-lecture mathematics class. The role of full-time facilitator was new for me and the role of exploratory learner was new for them. There was resistance and confusion. Most of the students were used to being receptacles into which definitions and theorems were dumped. Homework was typically solved by using the recipe shown in examples from lecture. As one student put it, “I usually just solve the problem by pattern matching.”

In addition to the complications that developed due to a change in roles for both the students and myself, there were other complexities as well. Ensuring that students understood course goals and developing appropriate assessment tools were two major ones. My purpose in the next several pages is to share my experience in using a set of PBL materials for a college-level mathematics course. I will discuss the materials used as well as the challenges we, as a class, faced. I will also discuss the positive impact such a pedagogical style can have on future teachers.

4.1. Course Demographics

The course in which the PBL materials were implemented in place of day-to-day lecture was a capstone mathematics course for pre-service mathematics teachers. The course is the first in a capstone series required of undergraduate mathematics majors who wish to continue into a teacher licensure program. However, the class is open to other students and is typically taken by students majoring or minoring in mathematics. During the Fall 2011 10-week quarter, 14 students participated in the course, and all data were collected with Institutional Review Board approval.

4.2. Course Materials

The materials used for the course were created by Darryl Yong and Ben Sinwell with assistance from Bowen Kerins and had initially been implemented in the Summer 2008 PCMI Secondary School Teachers Program. The title of the 2008 materials is Applications of Algebra and Geometry to the Craft of Teaching. They are available to the public and can be found at http://mathforum.org/pcmi/hstp/resources/course2008.html. The materials are focused on learning mathematics by working problems in a collaborative setting. The core ideas, though not obvious to students during the first lesson, explore Gaussian integers and Eisenstein integers, and the first lesson provides the building blocks for these concepts.

The 2008 PCMI materials are composed of 14 lessons in worksheet form. These lessons are not stand-alone lessons as some textbook chapters can be. The lessons are deeply interconnected and must be done in order and as a set.
One cannot pick and choose lessons to incorporate into a course. The materials must be used as a whole in place of a textbook and lecture or not at all. The use of these materials as supplementary notes or in pieces would not make sense and would cause content to be disjointed.

The materials are meant to be worked on in small groups. This approach does seem to be optimal, as students then gain multiple approaches and perspectives for each problem. As they did this, they made notes in the margins of their worksheets to which they often referred when working on later lessons. The basic idea is that students approach each lesson with the mathematics they already know and develop new ideas and connect concepts as they work. As one student put it:

I do not necessarily see the handout as providing any new information, but is rather the prompt for group activity, which will lead to the learning.

The goal is to develop new knowledge as they work through problems with their peers.

4.3. A New Learning Environment

As I stated earlier, most of the students’ previous mathematics courses had been lecture based and included traditional assessments (homework, quizzes, and exams). After conversations with students, I discovered that they had anticipated the same for the capstone course. Thus, the new learning environment provided new challenges. We each had new roles in the classroom, which required a shift in the way in which we approached the teaching and learning of mathematics.

4.3.1. New Roles

The students’ new roles were as active participants in knowledge building, not as passive listeners. Students were a bit surprised on the first day of class when they encountered the unfamiliar classroom format:

When coming into this class, I was expecting it to be another lecture math class where we just took notes and then were left to our own devices to really understand the material.

This new role took some adjustment on the part of the students, and they had obstacles to overcome while going through this process.

My role was significantly different as well. No longer would my time be spent primarily at the board discussing definitions, theorems, and examples. My duty was to facilitate learning. This posed its own challenges as well.
Students become developers of knowledge: Rather than taking their usual place in the mathematics classroom as receptacles of knowledge, the students in my class were to become developers of knowledge. They now shouldered much of the responsibility of developing mathematical ideas and making connections. The students adapted to working collaboratively on problems. However, this new way of learning was unfamiliar territory for most of them which sometimes lead to frustrations, as they did not have immediate access to solutions nor were they told what steps to follow if they got stuck. They had to figure it out as a team.

Their biggest struggles were. (i) identifying the intended goals of the lessons; (ii) knowing when they had met the intended goals; and (iii) connecting ideas and concepts between lessons. This is evident in their journal entries:

The lessons so far have been going great. I just feel lost and don’t know the objective.

This lesson was unclear on its goals, unlike the previous lessons so that made it less effective and more of just work rather than learning.

There was lots of information and ideas being presented and I had trouble sorting out how they were connected.

This self-discovery class based learning was going great but I feel I’m just missing information to make all the connections.

I feel like the structure was unclear. I wasn’t able to tell what the lesson goals were.

I did not like that the box problem was so broad. We spent a lot of time trying to find patterns before moving on to the questions and didn’t get through the whole lesson. I think some guiding questions to help find more patterns would be helpful.

Students now had to assume more of the responsibility in the learning process. Rather than being told how things worked, they had to develop ideas and make connections between mathematical concepts. In addition, the goals of each lesson were not explicitly stated. This created a great deal of uncertainty and doubt.

Students struggled with identifying key concepts and ideas. They had been so used to being told what they were supposed to learn and what skills they were supposed to develop. They were uncomfortable with uncertainty. However, as Zaslavsky [11, p.~297] asserts, uncertainty evokes “meaningful learning and foster[s] mathematical understanding.” The students were not used to being in a constructivist classroom. They were not accustomed to
making their own meaning and it took time for them to let go of the desire of wanting to be told what to do and how to do it.

However, working through problems that involve uncertainty and then reflecting upon the process lends itself to a better understanding of the mathematics involved as well as one’s own learning process [11]. Though it took time, students did adjust to this process and they were constantly reflecting on both current and previous work, as the content of the lessons were intertwined. Students regularly looked back at their previous work to identify how they approached problems as well as how they could use previous ideas to solve current problems. A process of reflection, whether done deliberately or not, seems to be built into the lessons.

For example, lesson 8 caused students to reflect back on what they did in lesson 7. The problem tied together the ideas of plotting Eisenstein integers in the complex plane and points on isometric dot paper. In lesson 7 [7], two of the problems asked students to plot points \((a,b)\) on isometric dot paper and then plot numbers \(a + \omega b\) in the complex plane. Lesson 8 [7] then tied together ideas in the opening problem: shown in Figure 3.

As one student stated, after working on lesson 8:

I found it helpful that the problem in a box brought us back to lesson 7. I also thought it was helpful that we went from plotting points on isometric dot paper to plotting points on a complex plane. This helped me realize that if you plot the coefficients on an omega integer on isometric dot paper, the distance from the origin is the same as if you plot the omega integer on a complex plane. For example, \(9 + 4\omega\) is plotted as \(7 + 2\sqrt{3}\) on a complex plane, and when the point \((9,4)\) is plotted on isometric dot paper, the point is 7 units to the right and \(2\sqrt{3}\) units up.

Such introspective comments had become more common. Comments had evolved from “More explanation of . . . would have helped” or “Right now I’m not making any connections to the previous lesson or what the objective is,” to comments that were more thoughtful. In fact, I would contend that there is evidence of mathematical learning in these student statements:
I particularly enjoyed the relationship between the two, I think introducing it after repeatedly referring to Pythagorean Triples made the connection easy to grasp.” (Here the student is referring to the relationship between the norm of a complex number and Pythagorean triples).

[T] his lesson ended up dealing a lot with values of square roots and how they could be found using the multiples of specific irrational numbers. It also connected back to the idea that the norm or squares was equal to the square of norms and how we could use this fact to break apart norms that are raised to powers.

The questions led step by step and allowed me to track with it and think about connections and where the answers were coming from.

This process of reflection was powerful for students and the results aligned with what Zaslavsky [11] found - it made students aware of how they learned mathematics as well as identify where they struggle and have gaps in their knowledge. In addition, it seemed to help them solidify concepts and ideas as well as make connections. After some time the students became used to uncertainty and some even appreciated it.

Lecturer becomes Facilitator: It was not just the students that found the PBL experience challenging. I had uncertainty as well, and using this as a valuable tool for reflection was not something that I readily knew how to do. I also did not anticipate student confusion and frustration to be as prevalent as it was in the first day.

The initial resistance from the students made my role as facilitator difficult. Often the students just wanted to know what they were supposed to be learning. On multiple occasions students would say “Can you just tell me what I am supposed to be doing?” or “Can you just tell me how this is supposed to be solved?” This, however, contradicted what it meant for me to be a facilitator. I was not supposed to simply tell them the answer. They were supposed to discover it. When faced with such questions, which occurred primarily at the beginning of the term, I would respond with one or more of the following statements:

Have you talked with your group members about this problem?

Let’s discuss what the problem is asking of you. What ideas or concepts are present here?

What did we learn in the last lesson? Are any of those ideas or concepts present in this problem?

Helping students ease into their new classroom environment became easier as the class progressed and we began to embrace our uncertainties about the
course, the content, and our new environment. I did have a set of facilitator’s notes intended to help in this progression.

However, the facilitator’s notes only contained an overview of the mathematics needed to solve the problems. It did not contain recommendations for facilitation questions to ask students to help them through mathematical obstacles. Since this was my first time implementing PBL and utilizing the PCMI materials in a classroom setting, I had yet to develop my own set of facilitation questions for students. Just as Heaton [5] had found the teacher’s guide to be lacking in details, I was unable to completely rely on the facilitator’s notes that accompanied the lessons.

The issue was not that the facilitator’s notes were poorly written, but rather that one cannot anticipate all possible responses a student will have to the material. The relationships the students had with me as well as with each other were not simple and their exact reactions to the content could not be predicted. They had a range of abilities and brought different experiences to the table. Just as Heaton [5, p.95] describes her fourth graders as “real students with real growing and changing personalities and abilities that played a central role in my teaching”, my students were unique human beings, each developing their mathematical knowledge in their own way. The manner in which their mathematics evolved influenced my interactions with them. I do not believe that there exists a set of facilitator’s notes that can anticipate such interactions.

It was a challenge to grasp the material as the students perceived it, and it was difficult to anticipate what would give them trouble. The key was to turn the daily uncertainties and surprises into tools for both teaching and learning. As Heaton [5] and Zaslavsky [11] have found, uncertainty for both the student and teacher can lead to deeper understanding of both mathematics and the process of learning mathematics. After multiple experiences using the PCMI materials with pre-service teachers in a classroom setting, I have begun to create a personal set of notes that contain facilitating questions. These experiences have given me a better idea of the types of reactions students will have to the materials, how they will work through problems in a group setting, and how they will begin to connect mathematical ideas. I now have a better understanding of the pre-service teacher mathematical process as it relates the the PCMI materials.

4.4. What Class “Looked” Like

The 14 students were divided into groups of three or four and groups were changed periodically, always at the beginning of a new lesson. By the end of the term every student had worked with every other student on more than one occasion.

Students would push desks together so that they could easily discuss problems and exchange ideas. Occasionally, groups would utilize the chalk
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The classroom was noisy but productive. I walked around from group to group listening as they worked. If students were stuck, I would ask them questions or just talk to them about what they were doing. Often, as students would begin to explain things to me, they would realize what they needed to do to solve the problem. Additionally, if one group figured out a problem, I would encourage inter-group conversations. This was very effective and it gave the group explaining their ideas and solutions an opportunity to reflect on their work and solidify concepts.

It was not uncommon for students to walk over to each other’s desks asking about how they were solving problems and making statements such as:

- Oh wow! We didn’t think of doing it that way!
- How did you do that?

Students reflected on their peers’ ideas stating:

- The square problem was a good challenge. It was interesting to see how people interpreted the question differently.
- Talking with [her] validated the topics I was looking at.

These comments illustrate the discourse that occurred on a daily basis.

4.4.1. Assessment Tools

When implementing a new teaching style it is only natural to ask: How do I properly assess my students? Active participation by students was an obvious and easy way to make daily assessments. Additionally, students were graded on individual homework assignments, written papers, a journal, and a final presentation. No exams were given in the class. In this paper I will only discuss assessments relevant to the qualitative data that were collected; class sessions (participation), student journals, and final presentations.

Participation : Attendance was taken daily, but students were allowed to miss at most 2 days of class without penalty. However, just being physically present was not enough. Students had to actively participate in their groups, providing valuable contributions to the learning process. Getting students to do this, though, was never a problem. They were excited about the material and eager to work with their peers.

With one exception, the students strived to not miss class. They recognized that class time was precious. It was during that time that they would collaborate
to work through problems and develop ideas. The students knew that much could be gained from working together rather than alone, remarking:

After completing the problems, working with other students and sharing ideas I realized that often times it is more efficient to not only attack the problems head on, but to look at them from different angles.

and

Our group collaborated well; we each approached the task differently at first, but were able to incorporate each member's strategies and insights in some manner as we worked through the problems.

Students found that the time spent working together enriched their knowledge, which then enabled them to attack problems on their own. Rather than using one tactic, they had multiple approaches which they had learned from their peers that they could use to tackle homework assignments. Not surprisingly, the student who missed class frequently struggled on homework assignments and had difficulty identifying the important concepts from the lessons.

Journal: A journal was collected from each student at the end of the term. Students were required to write an entry about each lesson completed. Each entry had to reflect on the following five questions.

1. As you began this lesson, what did you think the goals of the lesson were?
2. After completing the lesson, what do you think the goals of the lesson were?
3. Did you find the structure of the lesson to be effective in meeting the goals of the lesson?
4. What, in particular, did you find effective about the lesson?
5. What did you not find effective about the lesson?

Journal entries provided insights into how the students felt about the class - primarily its structure and its content:

I am continuing to like the structure of the lesson and whenever we are stuck professor will push us along.

It’s frustrating to work through a lesson assuming that everything will wrap up nicely at the end, but that didn’t happen with this lesson. Usually when the lesson doesn’t tie together at the end, the connections are made in the following lesson. I don’t like waiting that long to finally understand what the lesson was trying to get at, especially when we wait from a Thursday to the following Tuesday.
I work well when there’s a lot of dialogue as I’m learning a concept. I also learn well when I am doing a problem myself (as opposed to seeing a professor write a problem out on the board). This lesson combined both of these components and was thus an effective way of learning for me.

I don’t always know what I’m supposed to be noticing in these problems. I’m not sure if I am overlooking the simple, obvious or just missing something.

I thought it would have been helpful if we made a list at the end (or maybe in the next class period) to review what we had learned or make conjectures that we could refer back to as needed.

Conversations with students and student writing (journals) indicated that they enjoyed working in groups and valued the various perspectives their peers had to offer. However, there were times when they struggled to get through all the material and had difficulty making connections. In addition, there were a few lessons that took more than one class session. The class met only on Tuesdays and Thursdays, so waiting from a Thursday to a Tuesday to wrap up a lesson could be disruptive in their thought processes.

The journals enabled me to identify both weak and strong aspects of the course. I have learned a great deal about where to provide a bit more scaffolding as where I can emphasize independent exploration. Finding this delicate balance is not easy, but, if I can do that, I know that I will become a more effective facilitator and the students will get more out of the content.

**Final Presentation:** Just as the written papers replaced midterm exams, a final presentation replaced a comprehensive final exam. During the last 2 weeks of class students gave 40-minute presentations in pairs. Each pair was to present a mathematics topic of their choice in the format of their choice. Topics could be at any level but materials for the lesson had to be provided, whether they were lecture notes, worksheets, or other handout. Topics had to be approved by me and each pair had to meet with me in advance to ensure that they were well prepared and had all they needed to successfully present.

Presentation topics included middle school mathematics, tilings, modular arithmetic and encryption, games and statistics, the Fibonacci sequence, and mental math tricks. The most surprising result from these presentations was that every group, save one, created a set of materials that were problem based in nature and used them as their presentation. They did not lecture on their content. Presenters divided the other students into groups and walked around acting as facilitators - asking probing questions to get students to think about the problems on which they were working.
It was never indicated that students should do their presentations in a particular way, but having spent a term as students in a PBL classroom it was a natural method for them to select. Each group created a well-developed set of materials that guided students through the respective topics. The materials contained relevant mathematical questions that built upon previous questions and knowledge and focused on a core set of mathematical concepts. In addition, students took on the role of facilitator well. Some were a little nervous since they were being graded, but they appeared at ease asking the other students thoughtful questions about the content.

It should be noted that the pair that did not do a presentation that mirrored the pedagogical style from class had the student that missed nearly half of the class sessions. It is of no surprise that the student would not feel comfortable with or thoroughly understand this instructional technique.

I have since conducted two more courses for pre-service teachers using PBL with PCMI materials. Subsequent experiences have gone more smoothly, as I learned much from this first experience. Overall, results have been positive - with students enjoying the content and structure. This was evident in class discussions I had with students, during which we discussed the use of the PCMI materials and PBL in the classroom. Additionally, students continue to do PBL-type final presentations. They note that there is more work involved but also more to be gained as they see unanticipated ideas, questions, and conversations arise. Students find that they glean new ideas from their peers’ presentations as well. They also express a readiness to implement such techniques in the classroom and have a better understanding of what issues might arise for their students if they use PBL.

5. DISCUSSION

As the students progressed through the course, their journal reflections and classroom comments changed. At the beginning of the term, many students were of the “Just give me the answer” mindset, but gradually students changed their approach to the class and the content. I began to hear more comments such as:

This question relates back to what we did last class.

I hadn’t thought to approach the problem in that way.

[T] his lesson shadowed lesson 11 while using a different type of integer.

Students began to recognize the importance of making connections with previous content as well as the insight that could be gained from their peers. The
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The following sequence of quotes from one particular student illustrates the change that took place as he evolved with the course:

I failed to see a real goal in this lesson.

I think it is fine to take that kind of time on something but it seemed like there was some ambiguity at the start as to the depth we were supposed to complete the table and look for patterns.

The other problems flowed very nicely and as I went through them they all built on the previous problems without me even knowing it.

The different graphs were a really good way to see what was going on and to come up with patterns and ideas about extending the problems to more general solutions.

I liked how each group was asked to present a problem on the board and share it with the class. I think it was a good way of getting everyone on the same page and going deeper into the connections being made.

I really liked the problem where we had to come up with our own way to find the approximation of the square root of seven. It allowed me to think about the previous problems and why things worked the way they did and what I could do to manipulate my own approximation.

The uncertainty and reflective process are indicative of what many students went through as they grappled with the idea of constructing ideas on their own and with each other. Additionally, through the PBL experience, students came to value working in groups. This not only provided an opportunity to experience PBL, but I would contend that some of the student quotes presented in this paper provide evidence of mathematical learning.

It was not just the students that had a valuable learning experience. I did as well. I have developed a new perspective with regard to teaching mathematics. Mathematics does not have to be demonstrated on a chalk board perfectly by a single individual. Mathematics can be experienced, with mistakes being made and shared with others so that mathematical learning can occur.

However, using PBL to accomplish a constructivist approach in a mathematics classroom is no easy task. It is difficult to develop questions for students that facilitate mathematical learning. I struggled a great deal with knowing how to phrase responses to questions from students that did not simply give the answer. Finding the right balance between saying too little and divulging too much certainly is an art, and I would argue that being a facilitator for PBL materials is not something learned from reading, rather it is learned from experience.
doing over and over. I believe that my pre-service teachers are better equipped to use PBL by having this experience and I do not doubt that when these teachers implement PBL in their own classrooms, they will struggle just as I did.

6. CONCLUSIONS

The implementation of a set of PBL materials created a learning environment that was different for myself as well as my students. Students became responsible for developing mathematical knowledge, while I acted as facilitator to help them through this process. Though these new classroom roles posed a challenge to both myself and my students, they gave us insight into the teaching and learning of mathematics.

The students, as future teachers, now have an understanding of this pedagogical style that goes beyond what they may have read in a textbook or journal article. They have experienced first-hand what it means to struggle through material on a path of discovery. This experience has added to their knowledge of PBL so that they may better implement PBL in their own classrooms in the future.

This instructional experience was informative for both myself and the students. It has enabled me to improve upon my teaching and work toward a more student-centered classroom for all of the courses I teach. I have since used these and other PCMI materials with pre-service teachers in multiple university settings, with each experience having similar results; students giving final presentations that incorporated PBL. These experiences have furthered my knowledge of PBL as well as improved upon my understanding of what pre-service teachers need. I am now in a better position to develop and implement a more rigorous study of the use of PBL with pre-service mathematics teachers through the PCMI materials. For future courses I plan to include the PBL experience with the PCMI materials and then follow up with the students after they have been teaching for 2–3 years. I would survey the students to see who is implementing PBL in his or her classroom, and, if possible, observe his or her use of PBL. In addition, this information will help inform how might better conduct PBL in my classroom for future teachers.

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BIOGRAPHICAL SKETCH

Mary E. Pilgrim received her education at Colorado State University in Fort Collins, CO and has been actively participating in mathematics education, particularly with pre-service K-12 teachers and Calculus instruction. She was a Visiting Professor at Oregon State University for the 2011–2012 academic year. Currently she is Assistant Professor in Mathematics Education in the Department of Mathematics at Colorado State University.