M 546 Intro to Nonlinear PDE's

1. Existence-Uniqueness for Elliptic BVP's

A. Survey of Hilbert Space Results

B. Weak Formulation of Elliptic BVP's

variational formulation of BVP's
Galerkine approximations

2. First Order Partial Differential Equations

A. Examples of Nonlinear Behavior in 1st order ODE's

direction fields and solution curves for ODE's
local existence
failure of uniqueness, hysteresis

B. PDE's of Order 1

1. Method of characteristics
    - Integral curves for vector fields
    - Characteristics for quasilinear pde's of order 1
        - a linear pde with constant coefficients
        - a linear pde with variable coefficients
        - semilinear and quasilinear problems
        - blowup of solutions; the gradient catastrophe

2. Nonlinear Conservation Laws
    - conservation laws
    - Shock type solutions
    - Selection Principles
    - Expansion Fans
    - Additional topics
        - loss of uniqueness, extra constraints
        - irreverisibility of weak solutions

3. Systems of Conservation Laws

3. Higher Order Nonlinear PDE's

A. Travelling Wave Solutions

Linear travelling waves
Burger's equation with diffusion; artificial viscosity solution
Burger's equation with dispersion = KdV eqn: Solitons

Burger's equation with diffusion and dispersion
Perturbation methods for NL equations

B. **Similarity Solutions**

linear diffusion
nonlinear diffusion
Porous medium equation: 1-d and n-d

C. **Reaction Diffusion Equations**

TW solutions for polynomial forcing
Stability of TW solutions
Chemical solute problem

D. **Free Boundary Problems**?