**Problems on Travelling Wave Solutions**

1. Find any travelling wave solutions to the so called ”modified KdV equation”.

$$\partial_t u(x,t) + 6u^2 \partial_x u(x,t) + \partial_{xxx} u(x,t) = 0$$

where $u,\ \partial_x u \ and \ \partial_{xx} u \to 0 \quad as \quad |x| \to \infty$

2. Find any travelling wave solutions to the elastic medium equation

$$\partial_{tt} u(x,t) = \partial_{xx} u(x,t) + \partial_x u(x,t)\, \partial_{xx} u(x,t) + \partial_{xxxx} u(x,t)$$

where $\partial_x u,\ \partial_{xx} u \ and \ \partial_{xxx} u \to 0 \quad as \quad |x| \to \infty$

3. Find any travelling wave solutions to the equation

$$\partial_t u(x,t) = \partial_{xx} u(x,t) + \sin u(x,t)$$

where $u,\ \partial_x u \ and \ \partial_{xx} u \to 0 \quad as \quad |x| \to \infty$

4. Determine whether there are any travelling wave solutions

$$s(x,t) = S(x-ct), \qquad i(x,t) = I(x-ct),$$

for the reaction diffusion system

$$\partial_t s(x,t) = -s(x,t)^2\, i(x,t)$$

$$\partial_t i(x,t) = \partial_{xx} i(x,t) + s(x,t)^2\, i(x,t) - \beta\, i(x,t)$$

where $S(z) \to 1,\ I(z) \to 0 \ as \ z \to +\infty, \quad S'(z), I'(z) \to 0 \ as \ z \to -\infty$

5. Analyze the travelling wave solutions for

$$\partial_t u(x,t) + u\, \partial_x u(x,t) - \alpha\, \partial_{xx} u(x,t) + \beta\, \partial_{xxx} u(x,t) = 0$$

by converting the nonlinear ODE to a dynamical system and looking for homoclinic or heteroclinic orbits.
Analyze the interaction between the parameters $\alpha \ and \ \beta$ by determining the relative magnitudes where oscillating solutions exist and for what values the solutions cease to oscillate.

6. Analyze the travelling wave solutions for

$$\partial_t u(x,t) + sgn(u)\,\partial_x u(x,t) + \beta\,\partial_{xxx} u(x,t) = 0$$

Suppose $u(x,t) = v(x - ct)$ where $v',\ v" \to 0 \quad but\ v(z) \to\ const \quad as\ |z| \to \infty$

7. Find any travelling wave solutions to the equation

$$\partial_t u(x,t) = \partial_{xx} u(x,t) + u(u-a)(u-1)$$

where $0 < a < 1.$

Show that the associated dynamical system has three critical points and that for wave speeds below a certain critical value, two of the critical points are stable foci which leads to TW solutions with both positive and negative values (which are nonphysical in some contexts). Show that for $c^2 > 4(a+1)$ the TW solutions assume only positive values and that the heteroclinic orbits of the dynamical system in this case are true TW solutions for the PDE.

8.Find any travelling wave solutions to the equation

$$\partial_t u(x,t) = \partial_{xx} u(x,t) + u(u-a)(u-b)(u-1)$$

where $0 < a < b < 1.$

Assume whatever conditions at infinity you need to get a TWS but examine particularly how the sign of c (the wave speed) is affected by the relative amounts of area contained under the loops of $f(u) = u(u-a)(u-b)(u-1)$.