

## Conservation Law Equations Problems

Find global weak solutions for the following initial value problems

$$1. \partial_t u(x, t) + u \partial_x u(x, t) = 0, \quad u(x, 0) = \left\{ \begin{array}{ll} 0 & \text{if } x < 0, \text{ or } x > 3 \\ 3 & \text{if } 0 < x < 1 \\ 2 & \text{if } 1 < x < 2 \\ 1 & \text{if } 2 < x < 3 \end{array} \right\}$$

$$2. \partial_t u(x, t) + u \partial_x u(x, t) = 0, \quad u(x, 0) = \left\{ \begin{array}{ll} 0 & \text{if } x < -3, \text{ or } x > 2 \\ 4 & \text{if } -3 < x < -2 \\ 1 & \text{if } -2 < x < 1 \\ 3 & \text{if } 1 < x < 2 \end{array} \right\}$$

$$3. \partial_t u(x, t) + u \partial_x u(x, t) = 0, \quad u(x, 0) = \left\{ \begin{array}{ll} 1 & \text{if } 1 < |x| < 2 \\ 4 & \text{if } |x| < 1 \\ 0 & \text{if } |x| > 2 \end{array} \right\}$$

$$4. \partial_t u(x, t) + u \partial_x u(x, t) = 0, \quad u(x, 0) = \left\{ \begin{array}{ll} 1 & \text{if } x < -1 \\ x^2 & \text{if } -1 < x < 0 \\ 0 & \text{if } x > 0 \end{array} \right\}$$

$$5. \partial_t u(x, t) + u \partial_x u(x, t) = 0, \quad u(x, 0) = \left\{ \begin{array}{ll} 0 & \text{if } 1 < |x| \\ 1+x & \text{if } -1 < x < 0 \\ 1-x & \text{if } 0 < x < 1 \end{array} \right\}$$

$$6. \partial_t u(x, t) + u \partial_x u(x, t) = 0, \quad u(x, 0) = \left\{ \begin{array}{ll} 0 & \text{if } 1 < |x| \\ -\alpha x & \text{if } -1 < x < 0 \\ -\beta x & \text{if } 0 < x < 1 \end{array} \right\} \quad \beta > \alpha > 0.$$

$$7. \partial_t u(x, t) + \frac{1}{u^2} \partial_x u(x, t) = 0, \quad u(x, 0) = \left\{ \begin{array}{ll} 0 & \text{if } 1 < |x| \\ 1-x^2 & \text{if } |x| < 1 \end{array} \right\}$$

$$8. \partial_t u(x, t) + u \partial_x u(x, t) = 0, \quad u(x, 0) = \left\{ \begin{array}{ll} 0 & \text{if } x < 0, \text{ or } 1 < x \\ \sqrt{1-x} & \text{if } 0 < x < 1 \end{array} \right\}$$

$$9. \partial_t u(x, t) + u \partial_x u(x, t) = 0, \quad u(x, 0) = \left\{ \begin{array}{lll} 0 & \text{if} & x < -2 \\ 1 - x & \text{if} & -2 < x < 0 \\ 1 & \text{if} & 0 < x \end{array} \right\}$$

$$10. \partial_t u(x, t) + u \partial_x u(x, t) = 0, \quad u(x, 0) = \left\{ \begin{array}{lll} x(1 - x) & \text{if} & 0 < x < 1 \\ 0 & \text{if} & x \notin (0, 1) \end{array} \right\}$$

Hint: This one is difficult. Use Maple as much as possible.