The Last Problem Set

1. Determine if there are any solutions of the form $u(x,t) = t^a f\left(\frac{x}{t^b}\right)$ *for*

$$\partial_t(u^m) - \partial_{xx}u(x,t) = 0 \quad x \in R,\ t > 0$$
$$u,\ \partial_x u \to 0 \quad as \quad |x| \to \infty$$

2. Consider the nonlinear problems

(a) $u^2\partial_t u - \partial_{xx}u(x,t) = 0 \quad x \in R,\ t > 0$

(b) $\partial_t u - u^3\partial_{xx}u(x,t) = 0 \quad x \in R,\ t > 0$

What is a similarity solution for these equations?
Determine if these equations have similarity solutions.
Do linear equations have similarity solutions?

3.Consider the nonlinear problems

(a) $\partial_t u - \partial_{xx}u(x,t) = u(2-u)(4-u) \qquad x \in R,\ t > 0$

(b) $\partial_t u - u^2\partial_x u - \partial_{xx}u(x,t) = 0 \quad x \in R,\ t > 0$

What is a travelling wave solution for these equations?
Determine if these equations have travelling wave solutions.
Do linear equations have travelling wave solutions?

4. Use an energy argument to show that there is at most one classical solution for

$$\partial_t u + u\partial_x u - \partial_{xx}u(x,t) = 0 \qquad 0 < x < 1,\ t > 0,$$
$$u(x,0) = f(x),$$
$$u(0,t) = 0 = u(1,t)$$

Hint: Suppose u and v are 2 classical solutions and $w = u - v$. *Show w satisfies*

$$\partial_t w + \partial_x(a(x,t)w) - \partial_{xx}w = 0 \qquad 0 < x < 1,\ t > 0,$$
$$w(x,0) = 0,$$
$$w(0,t) = 0 = w(1,t)$$

where

$$a(x,t) = \tfrac{1}{2}(u+v).$$

Then let $E(t) = \frac{1}{2}\int_0^1 w^2 dx.$

5. Show that

$$\begin{aligned} \partial_t u(x,t) - \partial_{xx} u(x,t) &= u(x,t)(1-u(x,t)), && 0 < x < 1,\ t > 0, \\ u(x,0) &= f(x), && 0 < x < 1, \\ \partial_x u(0,t) = 0 &= \partial_x u(1,t), && t > 0. \end{aligned}$$

has a solution that is global in t.