

Problem C M545 Due Friday November 19 (solve at least one of the two problems)

I. A Radon source is placed at one end of a thin, air-filled tube of length  $L$ . The opposite end of the tube is sealed against the passage of Radon and a Radon detector is placed at the sealed end. Initially the tube is free of Radon, and as Radon diffuses down the tube, some of the Radon escapes through the sides of the tube due to leakage. This situation can be modelled by

$$\begin{aligned} \partial_t C(x, t) &= D \partial_{xx} C(x, t) - \beta C(x, t), & 0 < x < L, \quad t > 0, \\ C(x, 0) &= 0, & 0 < x < L, \\ C(0, t) &= C_0 > 0, & t > 0, \\ \partial_x C(L, t) &= 0, & t > 0. \end{aligned}$$

Here  $C(x, t)$  denotes the concentration of Radon at  $(x, t)$ ,  
 $D$  denotes the diffusivity of Radon in air,  
and  $\beta$  denotes a positive constant related to the Radon leakage.

1. Find the solution  $C(x, t)$  and in particular, find a function for  $C(L, t)$  for  $t > 0$ .
2. Describe the asymptotic behavior of  $C(L, t)$  as  $t$  tends to infinity. How does the behavior in the case  $\beta > 0$  differ from the limit in the case  $\beta = 0$ ?

II Suppose  $u(r, \theta)$  satisfies

$$\begin{aligned} \nabla^2 u(r, \theta) &= 0 & 1 < r < 2, \quad -\pi < \theta < \pi, \\ u(2, \theta) &= \begin{cases} 1 & \text{if } \pi/4 < \theta < 3\pi/4 \\ 0 & \text{otherwise} \end{cases} \\ \partial_r u(1, \theta) &= 0. \end{aligned}$$

At what point on the inner boundary  $\{r = 1, |\theta| < \pi\}$  does the maximum value of  $u$  occur?