M 545 Homework Assignment B Due Monday Nov 8

1. Suppose $u = u(x,y)$ solves

$$\nabla^2 u(x,y) = 0 \qquad x \in R,\ y > 0$$

$$u(x,0) = \begin{cases} x(1-x) & if \quad 0 < x < 1 \\ (x-9)(10-x) & if \quad 9 < x < 10 \\ 0 & otherwise \end{cases}$$

Find $u(x,y)$ and determine the values of y for which $u(5,y)$ is increasing and for which values it is decreasing (do this graphically if you can). How would you explain what you find?

2. Suppose $u = u(x,t)$ solves

$$\partial_t u = D\,\partial_{xx} u + V\,\partial_x u \qquad x \in R,\ t > 0$$

$$u(x,0) = \begin{cases} 1 & if \quad 0 < x < 1 \\ 0 & otherwise \end{cases}$$

Find $u(x,t)$ and plot the solution for $D = 1$ for several values of V, both positive and negative.
For a fixed V, plot the solution for several values of D, both greater than 1 and less than 1 (but only positive D).
What is the influence of the parameters D and V on the solution to this problem?

3.Suppose $w = w(x,t)$ solves

$$\partial_t w = D\,\partial_{xx} w + V\,\partial_x w \qquad x \in (0,10),\ t > 0$$

$$u(x,0) = \begin{cases} 1 & if \quad 0 < x < 1 \\ 0 & otherwise \end{cases}$$

$$w(0,t) = w(10,t) = 0$$

Let $w(x,t) = e^{\alpha x} U(x,t)$ and find the equation satisfied by $U(x,t)$.
Next, choose α so that the equation for $U(x,t)$ contains no $\partial_x U$ term (but it will contain a U term)
Identify the eigenfunctions for the U-problem
Write down the system of ODE's for the time dependent coefficients in the eigenfunction expansion
Solve the ODE's to obtain $U(x,t)$
Find the corresponding solution for $w(x,t)$