Practise Problems with Matrices:

Multiply the matrix times the vector:

$$\begin{bmatrix} 8 & 2 & 8 \\ 10 & 8 & 8 \\ 10 & 0 & 7 \end{bmatrix}\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} = \qquad ans = \begin{bmatrix} 12 \\ 2 \\ 17 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 4 \\ 2 & 0 & 8 \\ 10 & 5 & 9 \end{bmatrix}\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} = \qquad ans = \begin{bmatrix} 3 \\ 10 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 10 & 5 \\ 7 & 7 & 6 \\ 9 & 1 & 9 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} = \qquad ans = \begin{bmatrix} 19 \\ 15 \\ 10 \end{bmatrix}$$

Multiply the matrices:

$$\begin{bmatrix} 7 & 10 & 5 \\ 7 & 7 & 6 \\ 9 & 1 & 9 \end{bmatrix}\begin{bmatrix} 8 & 2 & 8 \\ 10 & 8 & 8 \\ 10 & 0 & 7 \end{bmatrix} = \qquad ans = \begin{bmatrix} 206 & 94 & 171 \\ 186 & 70 & 154 \\ 172 & 26 & 143 \end{bmatrix}$$

$$\begin{bmatrix} 8 & 2 & 8 \\ 10 & 8 & 8 \\ 10 & 0 & 7 \end{bmatrix}\begin{bmatrix} 4 & -1 & 3 \\ -5 & 5 & 2 \\ -4 & 3 & -3 \end{bmatrix} = \qquad ans = \begin{bmatrix} -10 & 26 & 4 \\ -32 & 54 & 22 \\ 12 & 11 & 9 \end{bmatrix} =$$

$$\begin{bmatrix} 5 & 2 & 4 \\ -3 & -3 & -1 \\ -5 & -2 & -2 \end{bmatrix}\begin{bmatrix} -4 & 1 & 1 \\ 3 & -4 & 0 \\ 4 & -1 & 2 \end{bmatrix} = \qquad ans = \begin{bmatrix} 2 & -7 & 13 \\ -1 & 10 & -5 \\ 6 & 5 & -9 \end{bmatrix}$$

$$\begin{bmatrix} -4 & 1 & 1 \\ 3 & -4 & 0 \\ 4 & -1 & 2 \end{bmatrix}\begin{bmatrix} 5 & 2 & 4 \\ -3 & -3 & -1 \\ -5 & -2 & -2 \end{bmatrix} = \qquad ans = \begin{bmatrix} -28 & -13 & -19 \\ 27 & 18 & 16 \\ 13 & 7 & 13 \end{bmatrix}$$

For each of the following matrices, reduce to row echelon form, determine the rank and, if necessary, find a basis for the nullspace.

$\begin{bmatrix} 4 & 1 & 3 \\ -5 & 0 & 3 \\ -5 & 1 & -4 \end{bmatrix}$, rank:= 3 reduction to triangular form $\begin{bmatrix} 4 & 1 & 3 \\ 0 & 5 & 27 \\ 0 & 0 & -62 \end{bmatrix}$

$\begin{bmatrix} 3 & 0 & 1 \\ 3 & 1 & -1 \\ 2 & 0 & -1 \end{bmatrix}$, row echelon form: $\begin{bmatrix} 3 & 0 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & -\frac{5}{3} \end{bmatrix}$,,

$\begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 1 \\ -3 & -3 & -2 \end{bmatrix}$, , row echelon form: $\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -5 \\ 0 & 0 & -23 \end{bmatrix}$

$\begin{bmatrix} 3 & -1 & -1 \\ -3 & 2 & 3 \\ -1 & -2 & 1 \end{bmatrix}$, , row echelon form: $\begin{bmatrix} 3 & -1 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & \frac{16}{3} \end{bmatrix}$

$\begin{bmatrix} -2 & 1 & 2 \\ -2 & 2 & 2 \\ 3 & 0 & -3 \end{bmatrix}$, , row echelon form: $\begin{bmatrix} -2 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, , nullspace basis: $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

$\begin{bmatrix} -1 & 2 & 1 \\ 0 & 1 & -2 \\ 3 & -1 & -1 \end{bmatrix}$,, row echelon form: $\begin{bmatrix} -1 & 2 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 12 \end{bmatrix}$

$\begin{bmatrix} -1 & 2 & 1 \\ 0 & 1 & 1 \\ 3 & -1 & 2 \end{bmatrix}$, , row echelon form: $\begin{bmatrix} -1 & 2 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$, nullspace basis: $\begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$

$\begin{bmatrix} -1 & 0 & -1 \\ -3 & 1 & -3 \\ 1 & -3 & 2 \end{bmatrix}$, row echelon form: $\begin{bmatrix} -1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 3 & 3 & 3 \\ 3 & 1 & 3 \\ -1 & 0 & -1 \end{bmatrix}$, row echelon form: $\begin{bmatrix} 3 & 3 & 3 \\ 0 & -2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, nullspace basis: $\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

$\begin{bmatrix} -1 & 2 & 0 \\ 2 & 1 & 1 \\ -3 & -3 & 1 \end{bmatrix}$, row echelon form: $\begin{bmatrix} -1 & 2 & 0 \\ 0 & 5 & 1 \\ 0 & 0 & \frac{14}{5} \end{bmatrix}$,

$\begin{bmatrix} -1 & 0 & -1 \\ -3 & 1 & -3 \\ 1 & -3 & 1 \end{bmatrix}$, row echelon form: $\begin{bmatrix} -1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ nullspace basis: $\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$,

$\begin{bmatrix} -3 & 2 & 1 \\ 1 & 1 & -2 \\ 2 & -1 & -1 \end{bmatrix}$,, row echelon form: $\begin{bmatrix} -3 & 2 & 1 \\ 0 & \frac{5}{3} & -\frac{5}{3} \\ 0 & 0 & 0 \end{bmatrix}$, nullspace basis: $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

Decide if the following vectors are linearly independent. If they are dependent, find constants c_1, c_2, c_3 such that $c_1\vec{v}_1 + c_2\vec{v}_2 + c_3\vec{v}_3 = \vec{0}$

(a) $\vec{v}_1 = \begin{bmatrix} 1 \\ -3 \\ 1 \end{bmatrix}$ $\vec{v}_2 = \begin{bmatrix} -3 \\ 1 \\ -3 \end{bmatrix}$ $\vec{v}_3 = \begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix}$

(b) $\vec{v}_1 = \begin{bmatrix} -3 \\ 1 \\ 2 \end{bmatrix}$ $\vec{v}_2 = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$ $\vec{v}_3 = \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix}$

For each of the following matrices, reduce to reduced row echelon form, determine the rank and find a basis for the rowspace and the column space.

$\begin{bmatrix} 0 & 4 & 2 & 1 \\ 3 & 6 & 3 & 3 \\ 3 & 2 & 1 & 2 \end{bmatrix}$, row echelon form: $\begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{1}{2} & \frac{1}{4} \\ 0 & 0 & 0 & 0 \end{bmatrix}$, rank = 2

row basis: $\left[\begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} \end{bmatrix}, \begin{bmatrix} 0 & 1 & \frac{1}{2} & \frac{1}{4} \end{bmatrix}\right]$,

column basis: $\left[\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\right]$

$\begin{bmatrix} 0 & 1 & -1 & -4 & 1 \\ 2 & 4 & 2 & 0 & 2 \\ 1 & 1 & 2 & 1 & 1 \end{bmatrix}$, row echelon form: $\begin{bmatrix} 1 & 0 & 3 & 0 & \frac{5}{3} \\ 0 & 1 & -1 & 0 & -\frac{1}{3} \\ 0 & 0 & 0 & 1 & -\frac{1}{3} \end{bmatrix}$, rank = 3

row basis: $\left[\begin{bmatrix} 1 & 0 & 3 & 0 & \frac{5}{3} \end{bmatrix}, \begin{bmatrix} 0 & 1 & -1 & 0 & -\frac{1}{3} \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 1 & -\frac{1}{3} \end{bmatrix}\right]$

, column basis: $\left[\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right]$

$\begin{bmatrix} -3 & 1 \\ 1 & -3 \end{bmatrix}$, eigenvectors: $\left\{\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right\} \leftrightarrow -2, \left\{\begin{bmatrix} -1 \\ 1 \end{bmatrix}\right\} \leftrightarrow -4,$