1. Under what conditions is $\sup(A)$ not an accumulation point for $A$?

If $a = \sup(A)$ is an isolated point of $A$ (in which case, it must belong to $A$)
For example $a = 3 = \sup\{1, 2, 3\}$ is not an accumulation point of the set.

2. Give an example of a set with the following properties or explain why no such set is possible:
   a) an infinite set with no accumulation points
      $\mathbb{N}$ the natural numbers has no acc pts
   b) a bounded set with no accumulation points
      $A = \{1, 2, 3\}$ is bounded and has no acc pts
   c) an interval $(a, b)$ containing only irrational numbers
      not possible, between any two real numbers $a < b$, there is an irrational
   d) a set $A \subset \mathbb{R}$ that contains its $\sup$ but not its $\inf$
      $(1, 2) \cup \{10\}$ contains its $\sup$, 10, but not its $\inf$, 1
   e) a finite set that does not contain its $\sup$
      not possible, the $\sup$ of a finite set is just the largest number in the set.

3. Show that every irrational number is an accumulation point of $\mathbb{R}$

Let $x =$ irrational number and let $\varepsilon > 0$ denote an arbitrary positive number. Then by the Archimedes principle, there exists a rational number, $r$, between $x - \varepsilon$ and $x + \varepsilon$.
Since $r$ is rational, it does not equal $x$ so $\forall \varepsilon > 0, \exists r, r \in \tilde{N}_\varepsilon(x)$. 
