

M545 PDE's Homework 4 (Due Friday, Oct. 31)

1. Determine which of the following functions are harmonic (show your work).

a) $x^3z - 3xy^2z + 6xyz^3$ b) $e^x \cos y$ c) $e^z \cos x \sin y$

2. Find the Green's function $G(x, y)$ for the 1-D equation

$$\frac{d}{dx} \left((x+1) \frac{du}{dx} \right) = f(x), \quad 0 < x < 1$$

$$u(0) = u(1) = 0$$

3. Show that the Fourier transform of $f(cx) = \frac{1}{|c|} \hat{f}(\frac{\xi}{c})$ and that the Fourier transform of $f(x)e^{iax}$ is $\hat{f}(\xi - a)$.

4. If $u_{xx} + u_{yy} = 0$ in $x^2 + y^2 < 1$ and $u = 3 + x + y$ on $x^2 + y^2 = 1$, find $u(\frac{1}{2}, \frac{1}{2})$.