MR2100302 (2006b:76049) 76M10 35Q35 65N55

Wang, Hong [Wang, Hong 9] (1-SC); **Liu, Jiangguo** (1-TXAM-IS); **Espedal, Magne S.** (N-BERG); **Ewing, Richard E.** (N-BERG)

A characteristic nonoverlapping domain decomposition method for multidimensional convection-diffusion equations. (English. English summary)

Numer. Methods Partial Differential Equations **21** (2005), *no.* 1, 89–103.

In this paper the authors present a domain decomposition method for the finite element approximation of the transient convection-diffusion equation based on the so-called Eulerian-Lagrangian localized adjoint method (ELLAM). Before describing the domain decomposition strategy proposed, the authors review the basic idea of this formulation as a basic flow solver.

The domain decomposition method is based on dividing the computational domain into a number of subdomains and applying the ELLAM formulation to each. The key point is the selection of the boundary conditions, which is done following the basic idea of the ELLAM method, that is, discretizing the total time derivative (the local time derivative plus the convective term) in a Lagrangian manner. For outflow boundaries, a Dirichlet type condition which is obtained by backtracking the value of the unknown along the characteristics of the velocity field is prescribed. Likewise, on the inflow boundary a Robin type condition is prescribed, again obtained from backtracking of the unknown.

The domain decomposition strategy presented can be used as an additive Schwarz preconditioner for the global problem. If A is the matrix of this problem and A_j the matrices obtained from the ELLAM method on each subdomain, $j = 1, \dots, J$, the preconditioner considered is $B = \sum_j I_j A_j^{-1} I_j^t$, where I_j is the injection from the discrete finite element space on a subdomain j into the global finite element space.

Another interesting observation made by the authors is that the domain decomposition presented can be viewed as a two grid strategy, the coarse grid being the one formed by the subdomains, in which the coarse grid solution can be trivially obtained. This is possible in cases that can be considered dominated by convection, with very small diffusion, since the backtracking of the unknown along the characteristics to obtain the boundary conditions for the problems in each subdomain already gives the coarse mesh solution.

Two simple numerical examples are presented in the paper, designed only to check the performance of the domain decomposition method. It is shown that, for a 2×2 partition of the unit square, this method gives results similar to the method without domain decomposition.

The main interest of the paper is the design of the transmission conditions on the interfaces of the subdomains, which is consistent with the underlying idea of the ELLAM method (in general, of methods based on the discretization along the characteristics). The use of this procedure as an additive Schwarz preconditioner is not exploited at all, and no results are reported about the usefulness of taking as the coarse grid solution the boundary conditions on the subdomains for convection dominated flows. The cost of the subdomain solver is not reported. *Ramon Codina* (E-UPB-RM)

MR2048246 (2004m:65135) 65M06 76M12

Younes, Anis

An accurate moving grid Eulerian Lagrangian localized adjoint method for solving the one-dimensional variable-coefficient ADE. (English. English summary)

Internat. J. Numer. Methods Fluids **45** (2004), *no.* 2, 157–178.

Summary: "An accurate finite-volume Eulerian Lagrangian localized adjoint method (ELLAM) is presented for solving the one-dimensional variable coefficient advection dispersion equation that governs transport of solute in a porous medium. The method uses a moving grid to define the solution and test functions. Consequently, the need for spatial interpolation, or equivalently numerical integration, which is a major issue in conventional ELLAM formulations, is avoided.

"After reviewing the one-dimensional method of ELLAM, we present our strategy and detailed calculations for both saturated and unsaturated porous media. Numerical results for a constant-coefficient problem and a variable-coefficient problem are very close to analytical and fine-grid solutions, respectively. The strength of the method developed is shown for a large range of CFL and grid Peclet numbers."

MR2022687 (2004m:65143) 65M50 65M12 65T60 76M25 76S05

Ewing, Richard E. (1-TXAM-IS); **Liu, Jiangguo** (1-TXAM-IS);
Wang, Hong [Wang, Hong 9] (1-SC)

Adaptive biorthogonal spline schemes for advection-reaction equations. (English. English summary)

J. Comput. Phys. **193** (2004), *no.* 1, 21–39.

In this article, the authors combine the Eulerian-Lagrangian localized adjoint method (ELLAM) with a spatial discretization based on biorthogonal spline wavelets to develop numerical schemes for multidimensional advection reaction equations. The idea is to use a weak formulation with test functions being solutions of the adjoint equation. These test functions are constructed using the method of characteristics and their point values are numerically obtained with a backward tracking algorithm. The discretization of trial and test functions in the weak formulation employs biorthogonal wavelets with primal scaling and wavelet functions being splines and thus having explicit expressions and known regularities. The use of multiresolution analysis allows one to set up a multilevel scheme with adaptive compression in which the wavelet coefficients below a certain threshold are suppressed. Numerical tests indicate that the presented schemes are second order accurate in space, first order accurate in time and unconditionally stable. The novelty of the work lies in the thresholding schemes used to adaptively reduce the numerical complexity in analogy with wavelet compression of data. *Michael Junk* (D-KNST-MS)

MR1994829 (2004f:65137) 65M25 65M60 76M25

Al-Lawatia, Mohamed (OM-SUQAS-MS)

An Eulerian Lagrangian localized adjoint methods for the solution of the transient advection diffusion equations in three space dimensions. (English. English summary)

Int. J. Pure Appl. Math. **7** (2003), *no.* 4, 435–447.

Summary: "We present a characteristic method for the solution of the transient advection diffusion equations in three space-dimensions. This method uses piecewise tri-linear trial and test functions over rectangular grids within the framework of the Eulerian Lagrangian localized adjoint methods (ELLAMs). It therefore maintains the advantages of previous ELLAM schemes. In particular, it treats general boundary conditions naturally in a systematic manner, conserves mass, and symmetrizes the governing transport equations. Moreover, it generates accurate numerical solutions even if large time steps are used in the simulation. Numerical experiments are presented to illustrate the

performance of this method."

MR1946802 (2003k:65104) 65M25 76M20 76S05

Al-Lawatia, Mohamed (OM-SUQA-MS); **Wang, Hong [Wang, Hong 9]** (1-SC)

A preliminary investigation on an ELLAM scheme for linear transport equations. (English. English summary)

Numer. Methods Partial Differential Equations **19** (2003), *no.* 1, 22–43.

In this paper the authors present numerical simulations in order to investigate the performance of the ELLAM (Euclidian-Lagrangian localized adjoint method) scheme. Firstly the authors formulate the ELLAM scheme for linear advection-reaction problems. Afterwards they compare the performance of the ELLAM with many other methods, including the MUSCL (monotonic upstream-centered scheme for consevation laws) scheme, the FCT (flux-corrected transport) scheme, the ENO (essentially non-oscillatory) scheme and the WENO (weighted ENO scheme). The test example is an incompressible flow in a 2-dimensional homogeneous medium. Finally the authors present observations and discussions. They give a very good interpretation of the numerical results. *Aziz Belmiloudi* (F-INSAR-LA)

MR1956016 (2004a:76091) 76M10 65M06 65M60 76S05 76T99

Wang, H. [Wang, Hong 9] (1-SC); **Liang, D.** (1-TXAM-IS); **Ewing, R. E.** (1-SC); **Lyons, S. L.**; **Qin, G.**

An ELLAM approximation for highly compressible multicomponent flows in porous media. (English. English summary)

Locally conservative numerical methods for flow in porous media.

Comput. Geosci. **6** (2002), *no.* 3-4, 227–251.

Summary: "We develop an ELLAM-MFEM approximation to the strongly coupled systems of time-dependent nonlinear partial differential equations (PDEs) and constraining equations, which describe fully miscible, highly compressible, multicomponent flows through heterogeneous and compressible porous media with singular sources and sinks. An Eulerian-Lagrangian localized adjoint method (ELLAM) is presented to solve the transport equations for concentrations. A mixed finite element method (MFEM) is used to solve the pressure PDE for the pressure and Darcy velocity simultaneously, which generates accurate fluid velocities and minimizes the numerical difficulties occurring in standard methods caused by differentiation of the

pressure and then multiplication by rough coefficients. The ELLAM-MFEM solution technique symmetrizes and stabilizes the governing transport PDEs and greatly reduces nonphysical oscillation and/or excessive numerical dispersion present in many large-scale simulators. Computational experiments show that the ELLAM-MFEM solution technique can generate stable and physically reasonable numerical simulations even if coarse spatial grids and very large time steps are used."

MR1911972 (2003g:65122) 65M60 65M06 76M25 76N99 76S05

Wang, H. [Wang, Hong 9] (1-SC)**; Zhao, W.** (1-SC)**;**
Ewing, R. E. (1-TXAM-IS)**; Lyons, S. L.; Qin, G.**

An ELLAM simulator for highly compressible flow in porous media with multiple wells. (English. English summary)

Fluid flow and transport in porous media: mathematical and numerical treatment (*South Hadley, MA*, 2001), 481–488, *Contemp. Math.*, 295, *Amer. Math. Soc.*, *Providence*, *RI*, 2002.

Summary: "We present an ELLAM-MFEM simulator for accurate, efficient, and robust simulations to highly compressible flows in porous media with multiple wells. An Eulerian-Lagrangian localized adjoint method (ELLAM) is used to solve the advection-diffusion transport partial differential equation for concentration, while a mixed finite element method (MFEM) is used to solve the pressure equation for the pressure and mass flow rate. The ELLAM-MFEM simulator symmetrizes the transport equation, and eliminates nonphysical oscillation and/or excessive numerical dispersion in many large-scale simulators in industrial applications.

"Computational experiments show that the ELLAM-MFEM simulator can accurately simulate incompressible and compressible flows in porous media with multiple and different types of wells, large mobility ratios, discontinuous permeabilities and porosities, and anisotropic dispersion in tensor form, even if very large time steps and spatial grids are used."

{For the entire collection see MR1911532 (2003b:76003)}

MR1911535 (2003g:65110) 65M25 76M25

Al-Lawatia, Mohamed (OM-SUQA-MS);
Wang, Hong [Wang, Hong 9] (1-SC)

A family of higher-order Eulerian-Lagrangian localized adjoint methods for advection-diffusion equations. (English. English summary)

Fluid flow and transport in porous media: mathematical and numerical treatment (*South Hadley, MA*, 2001), 25–35, *Contemp. Math.*, 295, *Amer. Math. Soc.*, *Providence*, *RI*, 2002.

The Euler-Lagrangian Localized Adjoint Methods (ELLAM) are applied to unsteady advection-diffusion problems. The numerical schemes are implemented using test and trial functions of linear-ELLAM, quadratic-ELLAM, and cubic-ELLAM types. The convergence rates of the higher-order schemes are given in [H. Wang, Numer. Methods Partial Differential Equations **14** (1998), no. 6, 739–780; MR1653338 (99i:65108)]. This paper is for the numerical justification of the theoretical results of [op. cit.].

{For the entire collection see MR1911532 (2003b:76003)}

Mohammad Asadzadeh (S-CHAL)

MR1911532 (2003b:76003) 76-06 00B25 65Mxx 76Mxx 76S05 76Txx

★**Fluid flow and transport in porous media: mathematical and numerical treatment.**

Proceedings of an AMS-IMS-SIAM Joint Summer Research Conference held at Mount Holyoke College, South Hadley, MA, June 17–21, 2001.

Edited by Zhangxin Chen and Richard E. Ewing.

Contemporary Mathematics, 295.

American Mathematical Society, *Providence*, *RI*, 2002. *x*+524 *pp.* $129.00. *ISBN* 0-8218-2807-X

Contents: Jørg Aarnes and Magne S. Espedal, A new approach to upscaling for two-phase flow in heterogeneous porous media (1–11); Clarisse Alboin, Jérôme Jaffré, Jean E. Roberts and Christophe Serres, Modeling fractures as interfaces for flow and transport in porous media (13–24); Mohamed Al-Lawatia and Hong Wang [Hong Wang[9]], A family of higher-order Eulerian-Lagrangian localized adjoint methods for advection-diffusion equations (25–35); César Almeida, Jim Douglas, Jr., Felipe Pereira, Luis Carlos Roman and Li-Ming Yeh, Algorithmic aspects of a locally conservative Eulerian-Lagrangian method for transport-dominated diffusive systems (37–48); Inga Berre, Helge K. Dahle, Kenneth H. Karlsen and Hans F. Nordhaug, A streamline front tracking method for two- and three-phase flow including capillary forces (49–61); Sandro Bitterlich and Peter Knabner, Adaptive and formfree identification of nonlinearities in fluid flow from column experiments (63–74); Alain Bourgeat, Overall behaviour of fractured porous media versus fractures' size and permeability ratio (75–92); Michael A. Celia and Andrew J. Guswa, Hysteresis and upscaling to two-phase flow through porous media (93–104); Benito M. Chen-Charpentier and Hristo V. Kojouharov, Simulation of biobarrier-protozoa interaction in porous media (105–112); Hongsen Chen, Zhangxin Chen, Guanren Huan and Zhongxiao Wang, Mixed discontinuous FE methods and their applications to two-phase flow in porous media (113–125); Zhangxin Chen, Yanli Cui and Qiaoyuan Jiang, Two-phase immiscible flow with the viscous drag in naturally fractured reservoirs (127–139).

Zhangxin Chen, Guanren Huan and Baoyan Li, Mixed finite element methods for multiphase flow in petroleum reservoirs with multiple wells (141–152); Craig C. Douglas, Gundolf Haase and Mohamed Iskandarani, An acceleration procedure for the spectral element ocean model formulation of the shallow water equations (153–158); Jim Douglas, Jr., Felipe Pereira and Li-Ming Yeh, Relations between phase mobilities and capillary pressures for two-phase flows in fractured media (159–171); Jim Douglas, Jr. and Anna M. Spagnuolo, Parameter estimates for high-level nuclear transport in fractured porous media (173–183); Dugald B. Duncan and Yiqi Qiu, Overlapping grids for welltest analysis (185–194); Richard E. Ewing, Upscaling of biological processes and multiphase flow in porous media (195–215); Richard E. Ewing, Junping Wang [Jun Ping Wang[1]], Suzanne L. Weekes and Yongjun Yang, A numerical simulation of multicomponent gas flow in porous media by projection methods (217–228); Xiaobing Feng, Recent developments on modeling and analysis of flow of miscible fluids in porous media (229–240); James Glimm, Yoon-ha Lee and Kenny Ye, A simple model for scale up error (241–251); James Glimm, Xiao Lin Li and Yingjie Liu, Conservative front tracking in one space dimension (253–264); Norbert Herrmann, BEM with collocation for the heat equation with Neumann and mixed boundary values (265–277).

Guanren Huan, Zhangxin Chen and Baoyan Li, Applications of the control volume function approximation method to reservoir simulations (279–291); Kenneth D. Jarman and Thomas F. Russell, Analysis of 1-D moment equations for immiscible flow (293–304); Daniel L. Kern, John J. Westman and Floyd B. Hanson, Locally optimal pumping and treatment rates in uncertain environments (305–315);

Do Y. Kwak, A general multigrid framework for a class of perturbed problems (317–325); Baoyan Li, Zhangxin Chen and Guanren Huan, Modeling horizontal wells using hybrid grids in reservoir simulations (327–341); Jichun Li, A multiblock mixed finite element method for 2D and 3D elliptic problems on mixed unstructured grids and its parallelization (343–353); W. Brent Lindquist, Network flow model studies and 3D pore structure (355–366); Qingjie Liu, Pingping Shen and Puhua Yang, Pore scale network modelling of gas slippage in tight porous media (367–375); Qingjie Liu, Jinxun Wang, Puhua Yang and Pingping Shen, The calculation of relative permeability by history matching and Beth network model [The calculation of relative permeability by history matching and Bethe network model] (377–386); Ali A. Merrikh, José L. Lage and Abdulmajeed A. Mohamad, Comparison between pore-level and porous medium models for natural convection in a non-homogeneous enclosure (387–396); Arunn Narasimhan and José L. Lage, New models for predicting temperature-dependent viscous effects on flow through porous media (397–408).

Gergina Pencheva and Ivan Yotov, Balancing domain decomposition for porous media flow in multiblock domains (409–419); Béatrice Rivière and Mary F. Wheeler, Non conforming methods for transport with nonlinear reaction (421–432); Louis F. Rossi, A high order Lagrangian scheme for flow through unsaturated porous media (433–444); Marcus Sarkis, Partition of unity coarse spaces (445–456); Sam Subbey, Mike Christie and Malcolm Sambridge, Uncertainty reduction in reservoir modeling (457–467); Hong Wang [Hong Wang[9]], Jiangguo Liu, Magne S. Espedal and Richard E. Ewing, A Eulerian-Lagrangian substructuring domain decomposition method for multi-dimensional, unsteady-state advection-diffusion equations (469–480); H. Wang [Hong Wang[9]], W. Zhao, R. E. Ewing, S. L. Lyons and G. Qin [Guan Qin], An ELLAM simulator for highly compressible flow in porous media with multiple wells (481–488); Li Wu and G. F. Pinder, Single-degree freedom collocation method using Hermite polynomials (489–499); Xijun Yu and Yonghong Wu, A Taylor-Galerkin finite element method for one-dimensional hyperbolic conservation laws (501–518); Wen Zhang [Wen Zhang[1]] and Ian Gladwell, Morphological evolution of a 3D array of particles under surface diffusion (519–524).

{Most of the papers are being reviewed individually.}

MR1902704 (2003b:65100) 65M60 65M25 76M10 76S05

Chen, Zhangxin (1-SMU)

Characteristic mixed discontinuous finite element methods for advection-dominated diffusion problems. (English. English summary)

Comput. Methods Appl. Mech. Engrg. **191** (2002), *no.* 23-24, 2509–2538.

It is known that advection-diffusion problems often present serious numerical difficulties. Standard finite element and finite difference methods usually exhibit some combination of non-physical oscillation and excessive numerical dispersion. Many numerical methods have been developed to overcome these difficulties. In this paper the authors utilize the modified method of characteristics (MMOC) (or Eulerian-Lagrangian method) and combine it with mixed discontinuous finite element methods to solve numerically time-dependent advection-dominated diffusion problems. Because of the Lagrangian nature of the advection term, the MMOC method treats this term by a characteristic tracking scheme and is suitable for advection-dominated problems.

Three characteristic mixed discontinuous finite elements are introduced. The first method is based on standard MMOC procedures. This method is simple to define and analyse, but fails to preserve an integral identity satisfied by the differential problem. The second method is formulated using the modified method of characteristics with adjusted advection (MMOCAA) and preserves the integral identity globally. The third method is defined in terms of a local Eulerian-Lagrangian technique and preserves the identity locally. The relationships of the methods to the MMOC, MMOCAA, ELLAM, and CMFEM procedures are described in detail. Stability and convergence properties are studied for all the methods; unconditionally stable results and sharp error estimates are established. Preliminary numerical results are presented to validate some of the error estimates obtained. These three methods not only preserve the conceptual and computational merits of both characteristic-based procedures and discontinuous finite element schemes, they also possess new features such as being more stable, being more accurate, and being able to handle the case where the diffusion coefficient is zero.

The ultimate goal of the author is to use these characteristic mixed methods for solving some practical problems like porous media flow problems utilizing local grid refinement and domain decomposition, where we can see the great advantages of these methods over traditional (conforming) Galerkin finite element methods.

H. K. Verma (Ludhiana)

MR1856234 (2002h:65145) 65M25 65M60 76M10 76S05

Wang, Hong [**Wang, Hong 9**] (1-SC);
Shi, Xiquan [**Shi, Xi Quan 2**] (PRC-DUT);
Ewing, Richard E. (1-TXAM-IS)

An ELLAM scheme for multidimensional advection-reaction equations and its optimal-order error estimate. (English. English summary)

SIAM J. Numer. Anal. **38** (2001), *no.* 6, 1846–1885 (*electronic*).

This paper examines an ELLAM (Euclidian-Lagrangian localized adjoint method) scheme for 2-dimensional advection-reaction problems (ARP).

The authors present a method developed in order to establish convergence results and an optimal-order L^2 error estimate.

The authors formulate the ELLAM scheme for the problem (ARP) in the case where the domain studied is a rectangular domain. The derived numerical scheme does not require that the CFL (Courant-Friedrichs-Lewy) condition hold.

The L^2 error estimate is obtained by using Boolean interpolation [see, e.g., F.-J. Delvos and W. J. Schempp, *Boolean methods in interpolation and approximation*, Longman Sci. Tech., Harlow, 1989; MR1088250 (92k:41003)] and by developing suitable techniques (the techniques used by the authors in the 1-dimensional ELLAM scheme are not valid in the 2-dimensional case because they depend on the injection of the Sobolev space H^1 into the space of all continuous functions).

To validate the theoretical estimate, some numerical results are discussed.

Aziz Belmiloudi (F-INSAR-LA)

MR1820884 (2002a:65003) 65-02 65Mxx

Ewing, Richard E. (1-TXAM-IS);
Wang, Hong [Wang, Hong 9] (1-SC)

A summary of numerical methods for time-dependent advection-dominated partial differential equations. (English. English summary)

Numerical analysis 2000, Vol. VII, Partial differential equations.
J. Comput. Appl. Math. **128** (2001), *no.* 1-2, 423–445.

This is a brief survey of an enormous and active research area. Over 100 recent references are given. To show the applicability of the methods discussed, the paper introduces the parabolic advection-diffusion equations of miscible flow in a subsurface porous medium, and goes on to introduce the equations of two-phase immiscible displacement. Then, a variety of computational methods are discussed in the context of simplified partial differential equations including (hyperbolic) problems with no diffusion. The concentration in the paper is on designing methods which do not introduce oscillations, which have limited numerical diffusion, which represent shocks well without too much smearing, and which conserve mass locally or globally. In addition to considering finite-difference methods and upwinding, the paper considers Galerkin and Petrov-Galerkin finite element methods and relates them to the finite-difference methods. Then, it moves on to more recent developments. First, Eulerian methods are considered, including streamline diffusion finite elements (SDFEM) and total variation diminishing methods (TVD) including flux limiters and slope limiters, essentially non-oscillatory (ENO) schemes and their weighted extension (WENO) schemes, and the discontinuous Galerkin method (DG). Next, characteristic-based methods are considered, including the classical approach, the modified method of characteristics (MMOC) and its extension with adjusted advection (MMOCAA). This section continues with a discussion of Eulerian-Lagrangian localized adjoint methods (ELLAM) and characteristic mixed finite element methods (CMFEM). The discussion concentrates on the practical advantages of the various methods discussing the finer mathematical detail only where it is necessary to make comparisons or to understand how a method is implemented.

{For the entire collection see MR1822459 (2001j:65007)}

Ian Gladwell (1-SMU)

MR1876027 76M25

Wang, Hong [Wang, Hong 9] (1-SC);
Al-Lawatia, Mohamed (OM-SUQA-MS)

A comparison of ELLAM with ENO/WENO schemes for linear transport equations. (English. English summary)

Numerical treatment of multiphase flows in porous media (*Beijing*, 1999), 311–323, *Lecture Notes in Phys.*, 552, *Springer*, *Berlin*, 2000.

MR1876005 (2002g:76115) 76S05 76-06 76Mxx 76T99

★**Numerical treatment of multiphase flows in porous media.**

Proceedings of the International Workshop held in Beijing, August 2–6, 1999.

Edited by Zhangxin Chen, Richard E. Ewing and Zhong-Ci Shi.

Lecture Notes in Physics, 552.

Springer-Verlag, *Berlin*, 2000. *xxii*+445 *pp.* $86.00.
ISBN 3-540-67566-3

Contents: Zhangxin Chen and Richard E. Ewing, Mathematical and numerical techniques in energy and environmental modeling (1–21); Clarisse Alboin, Jérôme Jaffré, Jean E. Roberts, Xuewen Wang and Christophe Serres, Domain decomposition for some transmission problems in flow in porous media (22–34); Todd Arbogast, Numerical subgrid upscaling of two-phase flow in porous media (35–49); Peter Bastian, Zhangxin Chen, Richard E. Ewing, Rainer Helmig, Hartmut Jakobs and Volker Reichenberger, Numerical simulation of multiphase flow in fractured porous media (50–68); Aijie Cheng and Gaohong Wang, The modified method of characteristics for compressible flow in porous media (69–79); Hongsen Chen, Richard E. Ewing, Stephen L. Lyons, Guan Qin, Tong Sun and David P. Yale, A numerical algorithm for single phase fluid flow in elastic porous media (80–92); Tatiana Chernogorova [T. P. Chernogorova], Richard E. Ewing, Oleg Iliev and Raytcho Lazarov, On the discretization of interface problems with perfect and imperfect contact (93–103); Xia Cui, Finite element analysis for pseudo hyperbolic integral-differential equations (104–115); Ronald A. DeVore, Hong Wang [Hong Wang[9]], Jiang-Guo Liu and Hong Xu [Hong Xu[2]], A CFL-free explicit scheme with compression for linear hyperbolic equations (116–123); Craig C. Douglas, Jonathan Hu, Mohamed Iskandarani, Markus Kowarschik, Ulrich Rüde and Christian Weiss, Maximizing cache memory usage for multigrid algorithms for applications of fluid flow in porous media (124–137).

Jim Douglas, Jr., Felipe Pereira and Li-Ming Yeh, A locally conservative Eulerian-Lagrangian method for flow in a porous medium of a mixture of two components having different densities (138–155); Vladimir A. Garanzha, Vladimir N. Konshin, Stephen L. Lyons, Dimitrios V. Papavassiliou and Guan Qin, Validation of non-Darcy well models using direct numerical simulation (156–169); Norbert Herrmann, Mathematical treatment of diffusion processes of gases and fluids in porous media (170–178); Chieh-Sen Huang and Anna M. Spagnuolo, Implementation of a locally conservative Eulerian-Lagrangian method applied to nuclear contaminant transport (179–189); Xiuren Lei and Hong Peng [Hong Peng[1]], Application of a class of nonstationary iterative methods to flow problems (190–194); Baoyan Li and Yuanle Ma, Reservoir thermal recover simulation on parallel computers (195–207); Yuanxiang Li, Shengwu Xiong and Xiufen Zou, A class of lattice Boltzmann models with the energy equation (208–215); Zhibo Ma and Jianshi Zhu, Block implicit computation of flow field in solid rocket ramjets (216–221); Pingbing Ming and Zhong-Ci Shi, Stable conforming and nonconforming finite element methods for the non-Newtonian flow (222–231); Guan Qin, Hong Wang [Hong Wang[9]], Richard E. Ewing and Magne S. Espedal, Numerical simulation of compositional fluid flow in porous media (232–243).

Hilde Reme, Geir Åge Øye, Magne S. Espedal and Gunnar E. Fladmark, Parallelization of a compositional reservoir simulator (244–266); Thomas F. Russell, Relationships among some conservative discretization methods (267–282); Dongwoo Sheen, Parallel methods for solving time-dependent problems using the Fourier-Laplace transformation (283–291); Zhong-Ci Shi and Xuejun Xu, Cascadic multigrid methods for parabolic pressure problems (292–298); Sam Subbey and Jan-Erik Nordtvedt, Estimation in the presence of outliers: the capillary pressure case (299–310); Hong Wang [Hong Wang[9]] and Mohamed Al-Lawatia, A comparison of ELLAM with ENO/WENO schemes for linear transport equations (311–323); Hong Wang [Hong Wang[9]], Dong Liang, Richard E. Ewing, Stephen L. Lyons and Guan Qin, An accurate approximation to compressible flow in porous media with wells (324–332); Yan-Fei Wang and Ting-Yan Xiao, Fast convergent algorithms for solving 2D integral equations of the first kind (333–344); Ziting Wang and Xianggui Li, A two-grid finite difference method for nonlinear parabolic equations (345–350); Francisco R. Villatoro and Jesús García-Lafuente, A compact operator method for the omega equation (351–361).

Danping Yang, Domain decomposition algorithm for a new characteristic mixed finite element method for compressible miscible displacement (362–372); Dequan Yang, Tigui Fan and Xinyu Yang, A boundary element method for viscous flow on multi-connected domains (373–377); Xi-Jun Yu and Yonghong Wu, A characteristic difference method for 2D nonlinear convection-diffusion problems (378–389); Yirang Yuan, Fractional step methods for compressible multicomponent flow in porous media (390–403); Guoyou Zhang, Tigui Fan, Zhongsheng Zhao and Dequan Yang, A model and its solution method for a generalized unsteady seepage flow problem (404–408); Huaiyu Zhang and Jiachang Sun, Domain decomposition preconditioners for non-selfconjugate second order elliptic problems (409–418); Wen Zhang and Ian Gladwell, Performance of MOL for surface motion driven by a Laplacian of curvature (419–429); Weidong Zhao [Wei Dong Zhao], A high-order upwind method for convection-diffusion equations with the Newmann boundary condition (430–440).

{The papers will not be reviewed individually.}

MR1842206 65M60 76S05

Wang, Hong [Wang, Hong 9] (1-SC)

An ELLAM scheme for porous medium flows. (English. English summary)

Discontinuous Galerkin methods (*Newport*, *RI*, 1999), 445–450, *Lect. Notes Comput. Sci. Eng.*, 11, *Springer*, *Berlin*, 2000.

MR1842160 (2002b:65004) 65-06 65M60 65N30

★**Discontinuous Galerkin methods.**

Theory, computation and applications.

Papers from the 1st International Symposium held in Newport, RI, May 24–26, 1999.

Edited by Bernardo Cockburn, George E. Karniadakis and Chi-Wang Shu.

Lecture Notes in Computational Science and Engineering, 11.

Springer-Verlag, *Berlin*, 2000. *xii*+470 *pp.* $89.95.

ISBN 3-540-66787-3

Contents: Bernardo Cockburn, George E. Karniadakis and Chi-Wang Shu, The development of discontinuous Galerkin methods (3–50); Harold L. Atkins, Steps toward a robust high-order simulation tool for aerospace applications (53–61); Timothy J. Barth, Simplified discontinuous Galerkin methods for systems of conservation laws with convex extension (63–75); F. Bassi [Francesco Bassi] and S. Rebay, A high order discontinuous Galerkin method for compressible turbulent flows (77–88); Douglas N. Arnold, Franco Brezzi, Bernardo Cockburn and Donatella Marini, Discontinuous Galerkin methods for elliptic problems (89–101); Richard S. Falk, Analysis of finite element methods for linear hyperbolic problems (103–112); J. E. Flaherty, R. M. Loy, M. S. Shephard and J. D. Teresco, Software for the parallel adaptive solution of conservation laws by discontinuous Galerkin methods (113–123); Pierre A. Gremaud and John V. Matthews, Simulation of gravity flow of granular materials in silos (125–134); Thomas J. R. Hughes, Gerald Engel, Luca Mazzei and Mats G. Larson, A comparison of discontinuous and continuous Galerkin methods based on error estimates, conservation, robustness and efficiency (135–146); Bernardo Cockburn, Joseph W. Jerome and Chi-Wang Shu, The utility of modeling and simulation in determining transport performance properties of semiconductors (147–156).

Ohannes Karakashian and Theodoros Katsaounis, A discontinuous Galerkin method for the incompressible Navier-Stokes equations (157–166); Qun Lin [Qun Lin[1]], Full convergence for hyperbolic finite elements (167–177); J. Tinsley Oden and Carlos Erik Baumann, A conservative DGM for convection-diffusion and Navier-Stokes problems (179–196); F. Bassi [Francesco Bassi] and S. Rebay, GMRES discontinuous Galerkin solution of the compressible Navier-Stokes equations (197–208); Richard S. Falk and Gerard R. Richter, Explicit finite element methods for linear hyperbolic systems (209–219); Endre Süli, Christoph Schwab and Paul Houston, *hp*-DGFEM for partial differential equations with nonnegative characteristic form (221–230); Béatrice Rivière and Mary F. Wheeler, A discontinuous Galerkin method applied to nonlinear parabolic equations (231–244); Dan Aharoni and Amnon Barak, Parallel iterative discontinuous Galerkin finite-element methods (247–254); Steeve Augoula and Rémi Abgrall, A discontinuous projection algorithm for Hamilton Jacobi equations (255–261); Arjen C. B. Bogaerds, Wilco M. H. Verbeeten and Frank P. T. Baaijens, Successes and failures of discontinuous Galerkin methods in viscoelastic fluid analysis (263–270).

Wei Cai, High order current basis functions for electromagnetic scattering of curved surfaces (271–276); F. L. Carranza and R. B. Haber, An adaptive discontinuous Galerkin model for coupled viscoplastic crack growth and chemical transport (277–283); Paul Castillo, An optimal estimate for the local discontinuous Galerkin method (285–290); Bernardo Cockburn, Mitchell Luskin, Chi-Wang Shu and Endre Süli, Post-processing of Galerkin methods for hyperbolic problems (291–300); Nicholas Coult, Introduction to discontinuous wavelets (301–308); Clint Dawson, Vadym Aizinger and Bernardo Cockburn, The local discontinuous Galerkin method for contaminant transport problems (309–314); Bruno Despres, Discontinuous Galerkin method in the numerical solution of Euler equations in axisymmetric geometry (315–320); A. Fortin, A. Béliveau, M. C. Heuzey and A. Lioret, Ten years using discontinuous Galerkin methods for polymer processing problems (321–326); Donald J. Estep and Roland W. Freund, Using Krylov-subspace iterations in discontinuous Galerkin methods for nonlinear reaction-diffusion systems (327–335); John Greenstadt, An abridged history of cell discretization (337–342).

Changqing Hu, Olga Lepsky and Chi-Wang Shu, The effect of the least square procedure for discontinuous Galerkin methods for Hamilton-Jacobi equations (343–348); Guido Kanschat and Franz-Theo Suttmeier, A posteriori error estimate in the case of insufficient regularity of the discrete space (349–354); David A. Kopriva, Stephen L. Woodruff and M. Y. Hussaini, Discontinuous spectral element approximation of Maxwell's equations (355–361); Mats G. Larson and Timothy J. Barth, A posteriori error estimation for adaptive discontinuous Galerkin approximations of hyperbolic systems (363–368); Jian-Guo Liu and Chi-Wang Shu, A numerical example on the performance of high order discontinuous Galerkin method for 2D incompressible flows (369–374); I. Lomtev, R. M. Kirby and G. E. Karniadakis, A discontinuous Galerkin method in moving domains (375–383); Robert B. Lowrie and Jim E. Morel, Discontinuous Galerkin for hyperbolic systems with stiff relaxation (385–390); Luc Machiels, Finite element output bounds for parabolic equations: application to heat conduction problems (391–396); Manoj K. Prasad, Jose L. Milovich, Aleksei I. Shestakov, David S. Kershaw and Michael J. Shaw, 3D unstructured mesh ALE hydrodynamics with the upwind discontinuous Galerkin method (397–405); P. Rasetarinera, M. Y. Hussaini and F. Q. Hu, Some remarks on the accuracy of a discontinuous Galerkin method (407–412).

Mirko Sardella, Coupling continuous and discontinuous techniques: an adaptive approach (413–418); Dirk Schwanenberg and Jürgen Köngeter, A discontinuous Galerkin method for the shallow water equations with source terms (419–424); Spencer Sherwin, Dispersion analysis of the continuous and discontinuous Galerkin formulations (425–431); Howard Swann, The cell discretization algorithm; an overview (433–438); H. van der Ven and J. J. W. van der Vegt, Accuracy, resolution, and computational complexity of a discontinuous Galerkin finite element method (439–444); Hong Wang [Hong Wang[9]], An ELLAM scheme for porous medium flows (445–450); Tim Warburton, Application of the discontinuous Galerkin method to Maxwell's equations using unstructured polymorphic *hp*-finite elements (451–458); Lin Yin, Amit Acharya, Nahil Sobh, Robert B. Haber and

Daniel A. Tortorelli, A space-time discontinuous Galerkin method for elastodynamic analysis (459–464); Zhimin Zhang, Nonconforming, enhanced strain, and mixed finite element methods—a unified approach (465–470).

{Some of the papers are being reviewed individually.}

MR1780614 (2001g:76037) 76M10 65N30 76S05

Wang, Hong [Wang, Hong 9] (1-SC); **Liang, Dong** (PRC-SHAN-MSS); **Ewing, Richard E.** (1-TXAM-IS); **Lyons, Stephen L.**; **Qin, Guan**

An approximation to miscible fluid flows in porous media with point sources and sinks by an Eulerian-Lagrangian localized adjoint method and mixed finite element methods. (English. English summary)

SIAM J. Sci. Comput. **22** (2000), *no.* 2, 561–581 (*electronic*).

This paper presents an Eulerian-Lagrangian localized adjoint method (ELLAM)–mixed finite element method (MFEM) solution technique for accurate numerical simulation of coupled systems of partial differential equations, which describe complex fluid flow processes in porous media. An ELLAM, which was shown previously to outperform many widely used methods in the context of linear convection-diffusion PDEs, is presented to solve the transport equation for concentration. Since accurate fluid velocities are crucial in numerical simulations, an MFEM is used to solve the pressure equation for the pressure and Darcy velocity. This minimizes the numerical difficulties occurring in standard methods for approximating velocities caused by differentiation of the pressure and then multiplication by rough coefficients.

The ELLAM-MFEM solution technique significantly reduces temporal errors, symmetrizes the governing transport equation, eliminates nonphysical oscillation and/or excessive numerical dispersion in many simulators, conserves mass, and treats boundary conditions accurately. Numerical experiments show that the ELLAM-MFEM solution technique simulates miscible displacements of incompressible fluid flows in porous media accurately with fairly coarse spatial grids and very large time steps, which are one or two orders of magnitude larger than the time steps used in many methods. Moreover, the ELLAM-MFEM solution technique can treat large mobility ratios, discontinuous permeabilities and porosities, anisotropic dispersion in tensor form, and point sources and sinks. *Jian-Ping Zhu* (Akron, OH)

MR1756427 (2001e:65144) 65M15 65M25 76M10 76S05

Wang, Hong [Wang, Hong 9] (1-SC)

An optimal-order error estimate for an ELLAM scheme for two-dimensional linear advection-diffusion equations. (English. English summary)

SIAM J. Numer. Anal. **37** (2000), *no.* 4, 1338–1368 (*electronic*).

Let $\Omega=(x^L,x^R)\times(y^L,y^R)\subset\mathbf{R}^2$, $\Gamma=\partial\Omega$, $\boldsymbol{\nu}=$ the unit outward normal, $\Gamma'=\{x\in\Gamma\colon\ \mathbf{v}\cdot\boldsymbol{\nu}<0\}$, and $\Gamma^0=\{x\in\Gamma\colon\ \mathbf{v}\cdot\boldsymbol{\nu}>0\}$. The author treats the linear advection-diffusion PDE

$$c_t+\boldsymbol{\nabla}\cdot(\mathbf{v}(\mathbf{x},t)c-D(\mathbf{x},t)\boldsymbol{\nabla}c)=f(\mathbf{x},t),\quad \mathbf{x}\in\Omega,\ t\in[0,T],$$

with $(\mathbf{v}c-D\boldsymbol{\nabla}c)\cdot\boldsymbol{\nu}=g'(\mathbf{x},t)$ for $\mathbf{x}\in\Gamma'$, $c=g^0(\mathbf{x},t)$ for $\mathbf{x}\in\Gamma^0$, and $c(\mathbf{x},0)=c_0(\mathbf{x})$ for $x\in\Omega$. Let $\mathbf{x}_{kl}=(x^L+k\Delta x,y^L+l\Delta y)$, $t_n=n\Delta t$, satisfying $\mathbf{x}_{ij}=(x^R,y^R)$, $t_N=T$, and $\Omega_\Delta=\{\mathbf{x}_{kl}\}$, $I_\Delta=\{t_n\}$. He gives an ELLAM scheme using a test function w satisfying $w_t+\mathbf{v}\cdot\boldsymbol{\nabla}w=0$, and the solutions C of the form $C(\mathbf{x},t_n)=\sum_A C(\mathbf{x}_{ij},t_n)\varphi_{ij}(\mathbf{x})+\sum_B g^0(\mathbf{x}_{ij},t_n)\varphi_{ij}(\mathbf{x})$, where $\sum_A$ is the sum for $\mathbf{x}_{ij}\in\Omega_\Delta\setminus\Gamma^0$, $\sum_B$ is the sum for $\mathbf{x}_{ij}\in\Gamma^0$, and the $\varphi_{ij}(\mathbf{x})$ are basis functions satisfying $\varphi_{ij}(\mathbf{x}_{kl})=\delta_{ik}\delta_{jl}$. Under the assumptions on $D,\mathbf{v},f,c$ and c_t, he obtains an optimal-order C^2 error estimate of $\|C-c\|$ in $\hat{L}^\infty(0,T;L^2(\Omega))$ on $\Omega\times I_\Delta$, and an estimate of $\|C-c\|$ in $\hat{L}^\infty(0,T;\hat{H}^1(\Omega))$ on $\{\mathbf{x}_{k-1/2,l-1/2}\}\times I_\Delta$ for $c\colon I_\Delta\to H^3(\Omega)$.
Hideo Yamagata (Osaka)

MR1752620 (2000k:76129) 76S05 76M25

Wang, Hong [Wang, Hong 9] (1-SC); **Liang, Dong** (1-SC);
Ewing, Richard E. (1-TXAM-IS); **Lyons, Stephen L.**;
Qin, Guan

An ELLAM-MFEM solution technique for compressible fluid flows in porous media with point sources and sinks. (English. English summary)

J. Comput. Phys. **159** (2000), *no.* 2, 344–376.

Summary: "We develop an ELLAM-MFEM solution procedure for the numerical simulation of compressible fluid flows in porous media with point sources and sinks. An Eulerian-Lagrangian localized adjoint method (ELLAM), which was previously shown to outperform many widely used and well-regarded methods in the context of linear transport partial differential equations, is presented to solve the transport equation for concentration. Since accurate fluid velocities are crucial in numerical simulations, a mixed finite element method (MFEM) is used to simultaneously solve the pressure equation as a

system of first-order partial differential equations for the pressure and mass flow rate. This minimizes the numerical difficulties occurring in standard methods caused by differentiation of the pressure and then multiplication by rough coefficients.

"Computational experiments show that the ELLAM-MFEM solution procedure can accurately simulate compressible fluid flows in porous media with coarse spatial grids and very large time steps, which are one or two orders of magnitude larger than those used in many numerical methods. The ELLAM-MFEM solution technique symmetrizes the governing partial differential equations, and greatly reduces or eliminates non-physical oscillation and/or excessive numerical dispersion present in many large-scale simulators that are widely used in industrial applications. It conserves mass and treats boundary conditions in a natural manner. It can treat large adverse mobility ratios, discontinuous permeabilities and porosities, anisotropic dispersion in tensor form, compressible fluid, heterogeneous media, and point sources and sinks." 2000Academic Press

MR1703282 (2000d:65166) 65M25 76M10 76S05

Wang, Hong [Wang, Hong 9] (1-SC)**;**
Dahle, Helge K. (N-BERG)**; Ewing, Richard E.** (1-TXAM-IS)**;**
Espedal, Magne S. (N-BERG)**; Sharpley, Robert C.** (1-SC)**;**
Man, Shushuang (1-SC)

An ELLAM scheme for advection-diffusion equations in two dimensions. (English. English summary)

SIAM J. Sci. Comput. **20** (1999), *no.* 6, 2160–2194 (*electronic*).

In this paper, the authors discuss ELLAM, a class of characteristic methods for solving two-dimensional linear advection-diffusion equations with variable coefficients. The emphasis is on implementation details, including issues of boundary conditions. One- and two-dimensional examples are provided to illustrate the behavior of the proposed methods. *Chi-Wang Shu* (1-BRN-A)

MR1692563 (2000b:76086) 76M25 65M70

Wang, Hong [Wang, Hong 9] (1-SC); **Ewing, Richard E.** (1-TXAM-IS); **Qin, Guan**; **Lyons, Stephen L.**; **Al-Lawatia, Mohamed** (OM-SUQA-MS); **Man, Shushuang** (1-SC)

A family of Eulerian-Lagrangian localized adjoint methods for multi-dimensional advection-reaction equations. (English. English summary)

J. Comput. Phys. **152** (1999), *no.* 1, 120–163.

Summary: "We develop a family of Eulerian-Lagrangian localized adjoint methods for the solution of the initial-boundary value problems for first-order advection-reaction equations on general multi-dimensional domains. Different tracking algorithms, including the Euler and Runge-Kutta algorithms, are used. The derived schemes, which are fully mass conservative, naturally incorporate inflow boundary conditions into their formulations and do not need any artificial outflow boundary conditions. Moreover, they have regularly structured, well-conditioned, symmetric, and positive-definite coefficient matrices, which can be efficiently solved by the conjugate gradient method in an optimal order number of iterations without any preconditioning needed. Numerical results are presented to compare the performance of the ELLAM schemes with many well studied and widely used methods, including the upwind finite difference method, the Galerkin and the Petrov-Galerkin finite element methods with backward-Euler or Crank-Nicolson temporal discretization, the streamline diffusion finite element methods, the monotonic upstream-centered scheme for conservation laws, and the Minmod scheme."

MR1653338 (99i:65108) 65M60 65M12 76M10

Wang, Hong [Wang, Hong 9] (1-SC)

A family of ELLAM schemes for advection-diffusion-reaction equations and their convergence analyses. (English. English summary)

Numer. Methods Partial Differential Equations **14** (1998), *no.* 6, 739–780.

The author considers an advection-dominated parabolic second-order differential equation in one space variable. He constructs a mass-conserving Eulerian-Lagrangian localized adjoint method for its numerical solution, using space-time finite elements that are piecewise polynomials of degree $m \geq 1$ in the space direction. For Dirichlet, Neumann and flux inflow and outflow boundary conditions, he proves convergence of order $h^{m+1} + k$ (here h is the spatial mesh diameter, k the temporal mesh diameter) in discrete $L^\infty(L^2)$ and $L^2(H^1)$ norms. Numerical results for the case $m = 1$ are presented.

Martin Stynes (IRL-CORK)

MR1377256 (97a:65076) 65M15 65M12 65M25 76M25

Ewing, Richard E. (1-TXAM-IS);
Wang, Hong [Wang, Hong 9] (1-SC)

An optimal-order estimate for Eulerian-Lagrangian localized adjoint methods for variable-coefficient advection-reaction problems. (English. English summary)

SIAM J. Numer. Anal. **33** (1996), *no.* 1, 318–348.

In a previous paper, the authors developed the so-called Euler-Lagrangian localized adjoint method (ELLAM) for the numerical solution of advection-reaction problems [R. E. Ewing and H. Wang, Adv. Hydro-Science Engrg. (B) **1** (1993), 2010–2015; per bibl.]. It is the purpose of the present paper to derive an estimate for the convergence rate of this method.

For simplicity, the following initial-boundary value problem in one space dimension is studied by the authors: $u_t + (vu)_x + ku = f$ for $a < x < b$, $0 \le t \le T$, $u(t,a) = g(t)$, $u(0,x) = u_0(x)$ with given functions $v(t,x)$, $k(t,x)$, $f(t,x)$, $g(t)$ and $u_0(x)$. It is shown that the numerical error produced by ELLAM is of order 2 in dx and 1 in dt. Numerical examples given at the end of the paper confirm that these are optimal-order estimates.

Reinhard Redlinger (Eggenstein-Leopoldshafen)

MR1701441 65M70 76M25 76S05 80M25

Wang, Hong [Wang, Hong 9] (1-SC);
Ewing, Richard E. (1-TXAM-IS); **Sharpley, Robert C.** (1-SC)

On different ELLAM schemes for reactive transport equations. (English. English summary)

Advanced mathematics: *computations and applications* (*Novosibirsk*, 1995), 252–262, *NCC Publ.*, *Novosibirsk*, 1995.

MR1701419 (2000c:00028) 00B25 65-06

★**Advanced mathematics: computations and applications.**

Proceedings of the International Conference (AMCA-95) held in Novosibirsk, June 20–24, 1995.

Edited by Anatoly S. Alekseev and Nicolay S. Bakhvalov.

NCC Publisher, *Novosibirsk*, 1995. x+748 *pp.*

The 82 papers in this collection include the following: A. S. Alekseev, Some properties of the multidisciplinary problems of mathematical modelling (2–13); Owe Axelsson, A generalized conjugate gradient minimum residual method with variable preconditioners (14–25); N. S. Bakhvalov, Iterative methods for stiff elliptic problems (26–31); V. P. Dymnikov, Some problems of the theory of climate (32–48); Richard E. Ewing, Multiphase flows in porous media (49–63); Yu. A. Kuznetsov [Yuriĭ Alekseevich Kuznetsov[1]], Iterative solvers for elliptic finite element problems on nonmatching grids (64–76); M. M. Lavrent′ev, Astronomical study of possibilities of transmission of information in the space-time (77–81); E. Magenes, Stefan problems in a concentrated capacity (82–90); G. I. Marchuk, Some problems of mathematical modelling in immunology and medicine (91–106); A. S. Sarkisyan and J. Suendermann, On one direction in world ocean mathematical modelling initiated by Academician G. Marchuk (107–116).

T. Scapolla, Numerical solution of Reissner-Mindlin plates with singular load using hierarchic high order finite elements (117–127); V. S. Vladimirov, p-adic numbers in mathematical physics (128–141); V. B. Barakhnin and G. S. Khakimzyanov, Adaptive-grid numerical solution of one-dimensional and two-dimensional problems for the shallow-water equations (144–153); V. M. Belolipetsky, The numerical simulation of hydrophysical processes in hydropower station bays (154–161); A. M. Blokhin and A. D. Birkin, Global resolving of some gas dynamics problems (162–166); E. V. Chizhonkov, On methods for solving the Stokes problem for a weakly-compressible and incompressible fluids (167–171); Yu. N. Grigoriev [Yu. N. Grigor′ev] and S. B. Meleshko [S. V. Meleshko], Group analysis of kinetic models (172–178); V. B. Karamyshev, V. M. Kovenya and S. G. Cherny [S. G. Chërnyĭ], The splitting-up method of solving multidimensional problems and its efficiency (179–194); G. S. Khakimzyanov, Adaptive-grid modelling of near-coast surface waves (195–203); D. V. Krasinsky, A. D. Rychkov and V. V. Salomatov, Numerical modelling of 3-D two-phase turbulent flows in a vortex furnace of boiler unit (204–211).

V. V. Ostapenko, Numerical modeling of wave flows in the Sarez lake caused by hazardous sliding of a lakeside landslide (212–217);

A. D. Rychkov, Numerical modeling of the interaction of sub- and supersonic turbulent two-phase jets with a plane obstacle (218–222); I. D. Sofronov and O. I. Sofronova, Parallelization of evolutionary problems in computational physics (223–228); Atusi Tani, On the localized induction equation for a vortex filament motion (229–239); Yu. N. Valitskiy [Yu. N. Valitskiĭ], Well-posedness of some problems with the given values of derivatives at several points (240–243); Hong Wang [Hong Wang[9]] and Richard E. Ewing, An Eulerian-Lagrangian localized adjoint method for second-order hyperbolic equations (244–251); Hong Wang [Hong Wang[9]], Richard E. Ewing and Robert C. Sharpley, On different ELLAM schemes for reactive transport equations (252–262); A. V. Avdeev, E. V. Goruynov and V. V. Skazka, On the numerical solution of the inverse problems in combined statements (264–272); G. Chavent, F. Clement, S. Gomez and B. Lavaud, Acoustic waveform inversion via Migration-Based TravelTime formulation: simple and complex structures (273–285); V. I. Kostin, V. G. Khajdukov [V. G. Khaĭdukov] and V. A. Tcheverda [V. A. Cheverda], On r-solutions of nonlinear equations (286–291).

V. G. Romanov, The structure of the initial-boundary value problem solution for the one-dimensional transport equation in the case of a point source (292–297); A. S. Zapreev, Some inverse problems and the integral equation of the convolution type (298–304); A. E. Aloyan, V. O. Arutyunyan, V. D. Egorov and V. A. Zagaynov, Numerical modeling of pollutants transport/photochemistry/aerosol formation in atmospheric disperse media (306–319); E. N. Golubeva, A seasonal cycle in a zonally averaged model of the thermohaline circulation (320–324); V. P. Kochergin, S. N. Sklyar and T. V. Dunets, Numerical algorithms for solving of hydrodynamic problems with boundary layers (330–336); V. V. Penenko, Methodology of inverse modeling for the problems of climate changes and environmental protection (358–367); V. V. Penenko, K. Wilderotter and M. S. Yudin, An adjoint equations approach for the simulation of the convection-diffusion flow with an application to a mesoscale meteorological model (368–373); S. N. Sklyar, A projective version of the integral interpolation method and its application for the discretization of the singular perturbation problems (380–385); M. S. Yudin, Numerical investigations of atmospheric motions over complex terrain (394–399); V. A. Barker and B. Gervang, On equal-degree spectral elements for the Stokes problem (402–412).

O. A. Doledenok and V. P. Il′in [Valeriĭ Pavlovich Il′in], On application of incomplete factorization methods for equations with variable coefficients (413–421); Yu. I. Kuznetsov, Complementary Jacobi matrix (422–427); M. Larin, Quadri-storage scheme for sparse Cholesky factor and tree-based implementation of Cholesky method (428–434); Y. Notay and Z. Ould Amar, Incomplete factorization preconditioning may lead to multigrid-like speed of convergence (435–446); Garret Sobczyk, A universal geometric algebra (447–455); V. N. Belykh, Overconvergence of numerical algorithms without saturation (on an example of elliptic problems) (458–462); V. B. Cherepennikov, Estimates of accuracy of approximate solutions of some linear systems of differential equations by the method of functional parameters (467–476); R. Z. Dautov, $O(h^2 \ln^\alpha 1/h)$ accuracy in distance for free boundaries in finite element approximation to the obstacle problems (477–490); V. I. Lebedev, Extremal polynomials with restrictions and optimal algorithms (491–502); P. M. Lima and A. C. Lemos, Finite element solution of degenerate boundary-value problems for ordinary differential equations (503–510); V. D. Liseikin, Survey of grid generation technology (511–517).

J. J. H. Miller, E. O'Riordan and G. I. Shishkin, On the use of fitted operator methods for singularly perturbed partial differential equations (518–531); V. V. Smelov, On demography hypothesis and a simple mathematical model of population development (532–538); V. S. Synakh, An approach to the spline collocation method (539–541); O. V. Ushakova, An efficient algorithm and program of generation of two-dimensional curvilinear optimal adaptive grids (542–549); V. V. Denisenko, The multigrid finite element variational method for the 2-D elliptic boundary value problems, that describe transfer processes with nonsymmetrical coefficients matrix (552–563); M. Kahlert, M. Paffrath and U. Wever, Comparison of data partitioning and domain decomposition methods in real life problems (564–576); Serguei Maliassov [S. Yu. Malyasov], A note on substructuring preconditioning for nonconforming finite element approximations of second order elliptic problems (577–587); V. V. Shaidurov, Cascadic algorithm with nested subspaces in domains with curvilinear boundary (588–595); T. A. Averina and S. M. Prigarin, Calculation of stochastic integrals of Wiener processes (598–605); Boris A. Kargin and Sergei M. Prigarin, Simulation of random fields in stochastic problems of the cloudy atmosphere optics (606–614); O. E. Krivosheev, A. M. Kolchuzhkin and I. S. Tropin, Investigation of polarized photon transport by Monte Carlo method (615–628).

V. S. Pugachev, I. N. Sinitsyn, E. R. Korepanov and V. I. Sinitsyn, Analytical research problems in stochastic differential systems (629–648); Yana L. Gurieva [Ya. L. Gur′eva] and Valery P. Il′in [Valeriĭ Pavlovich Il′in], On the finite-volume technology for mixed boundary-value problems (650–655); V. N. Kasyanov and V. K. Sabelfeld, Formal methods in program analysis and transformations (656–666); M. Katzouraki, M. Gergatsoulis and P. Kostarakis, An intelligent environment for solving elliptic partial differential equations (667–674); Paul N. Swarztrauber, The vector parallel paradigm (675–687); T. M. Yakhno, G. B. Cheblakov and V. Z. Zilberfain, Application of subdefinite calculations to the problems of computer design (688–698); J. P. Chabard, Turbulence modelling in finite element industrial applications (700–709); B. G. Mikhalenko and O. N. Soboleva, Numerical modeling of seismomagnetic effects in an elastic medium (722–730); M. V. Urev, Boundary conditions for Maxwell's equations for arbitrary time dependence (731–736); A. N. Yudin, EDS-3D: on computer technology of space-charge flow simulation in electromagnetic fields (737–746).

{Some of the papers are being reviewed individually.}

MR1343788 (96d:76070) 76M25 65M60 76M10 76R50 76S05

Dahle, Helge K. (N-BERG); **Ewing, Richard E.** (1-TXAM-IS); **Russell, Thomas F.** (1-COD)

Eulerian-Lagrangian localized adjoint methods for a nonlinear advection-diffusion equation. (English. English summary)

Comput. Methods Appl. Mech. Engrg. **122** (1995), *no.* 3-4, 223–250.

Summary: "Eulerian-Lagrangian localized adjoint methods (ELLAM) are developed for the nonlinear Buckley-Leverett equation, which is characterized by degenerate diffusion and sharpening near-shock solutions. The ELLAM methodology employs space-time finite elements with edges oriented along flow paths, and space-time test functions that satisfy a local adjoint condition. This combination extends Eulerian-Lagrangian concepts in a systematic mass-conservative fashion to problems with general boundary conditions. Various kinds of boundary conditions are considered, and a local time-stepping procedure is developed for a no-flow outlet condition that leads to a boundary layer. Numerical experiments illustrate the potential of these methods."

MR1342428 (96d:65171) 65M60 65M12 76M10 76Rxx
Wang, Hong [Wang, Hong 9] (1-SC);
Ewing, Richard E. (1-TXAM-IS); **Russell, Thomas F.** (1-COD)
Eulerian-Lagrangian localized adjoint methods for convection-diffusion equations and their convergence analysis. (English. English summary)
IMA J. Numer. Anal. **15** (1995), *no.* 3, 405–459.
The authors apply an Euler-Lagrangian localized adjoint method (ELLAM) to a time-dependent convection-diffusion problem in one space variable. They consider the mass-conservative implementation of this method with various boundary conditions. Error estimates are derived that are of optimal order when the diffusion coefficient is bounded away from zero. Numerical results substantiate the theory.

Martin Stynes (IRL-CORK)

MR1325395 (96b:65097) 65M25 76M10 76R99
Wang, Hong [Wang, Hong 9] (1-SC);
Ewing, Richard E. (1-TXAM-IS); **Celia, Michael A.** (1-PRIN-WT)
Eulerian-Lagrangian localized adjoint methods for reactive transport with biodegradation. (English. English summary)
Numer. Methods Partial Differential Equations **11** (1995), *no.* 3, 229–254.
This paper concerns numerical solution of 1D advection-reaction-diffusion equations and systems governing the degradation of organic contaminants.

At first, general Eulerian-Lagrangian localized methods (ELLAM) are developed for one single equation with constant coefficients. These schemes are adaptations of the standard Galerkin finite element method. There are two main differences: One is that the test functions are solutions of the homogeneous adjoint equation. This allows one to incorporate the relative sizes of advection, diffusion and reaction terms into the variational formulation. The other difference is that a characteristics technique is used to approximate the convective derivative. In the paper, some of the test functions are analytically obtained. Two different splittings of the adjoint equations lead to two possible sets of test functions. However, because of that splitting these functions do not take into account the global balance between advection, diffusion and reaction, but only of some combinations of two of them.

Next, the scheme for a single linear equation, including an explicit Euler discretization of the characteristics, is described. Moreover, a description of linearization strategies for solving nonlinear systems using ELLAM is given. Finally, some numerical experiments for significant test problems with mild nonlinearities are performed.

The main conclusion of the paper is that ELLAM techniques allow one to obtain accurate solutions for Péclet numbers much larger than standard methods. *Tomás Chacón Rebollo* (E-SEVL-ED)

MR1316373 (95m:65153) 65M12 65N12 76M25 76R99 76S05 86-08

Wang, Hong [Wang, Hong 9] (1-SC); **Ewing, R. E.** (1-TXAM-IS)

Optimal-order convergence rates for Eulerian-Lagrangian localized adjoint methods for reactive transport and contamination in groundwater. (English. English summary)

Numer. Methods Partial Differential Equations **11** (1995), *no.* 1, 1–31.

In this paper, a numerical analysis in dimension 1 for the Eulerian Lagrangian localized adjoint method (ELLAM), used for solving convection-diffusion-reaction equations, is carried out. This method is conservative and is well adapted for treating general boundary conditions. For a scalar one-dimensional convection-diffusion-reaction equation, optimal error estimates are given depending on spatial derivatives of the function and its total time derivative, but with no dependence on the partial time derivative. A superconvergence result is given. Numerical tests are also given, where the authors investigate separately the dependence of the convergence rates on the spatial mesh size and on the time step. *Yves Achdou* (F-POLY-AM)

MR1312370 (95i:65008) 65-06 00B25

★**Domain decomposition methods in scientific and engineering computing.**

Proceedings of the Seventh International Conference on Domain Decomposition held at the Pennsylvania State University, University Park, Pennsylvania, October 27–30, 1993.

Edited by David E. Keyes and Jinchao Xu.

Contemporary Mathematics, 180.

American Mathematical Society, *Providence*, *RI*, 1994. xxviii+546 *pp.* $77.00. *ISBN* 0-8218-5171-3

{The Sixth Conference has been reviewed [MR1262599 (94i:65004)].}

Contents: Folkmar A. Bornemann, Interpolation spaces and optimal multilevel preconditioners (3–8); Susanne C. Brenner, Two-level additive Schwarz preconditioners for nonconforming finite elements (9–14); H.-J. Bungartz, M. Griebel, D. Röschke and C. Zenger, Two proofs of convergence for the combination technique for the efficient solution of sparse grid problems (15–20); Xiao-Chuan Cai and Maksymilian Dryja, Domain decomposition methods for monotone nonlinear elliptic problems (21–27); Peter Deuflhard, Cascadic conjugate gradient methods for elliptic partial differential equations: algorithm and numerical results (29–42); Maksymilian Dryja, Multilevel methods for elliptic problems with discontinuous coefficients in three dimensions (43–47); Ralf Kornhuber and Harry Yserentant, Multilevel methods for elliptic problems on domains not resolved by the coarse grid (49–60); P. Le Tallec, T. Sassi and M. Vidrascu, Three-dimensional domain decomposition methods with nonmatching grids and unstructured coarse solvers (61–74); S. V. Nepomnyaschikh [S. V. Nepomnyashchikh], Domain decomposition for elliptic problems with large condition numbers (75–85); P. Oswald [Peter Oswald], Stable subspace splittings for Sobolev spaces and domain decomposition algorithms (87–98).

Luca F. Pavarino, A wire basket based method for spectral elements in three dimensions (99–104); Alex Pothen, An analysis of spectral graph partitioning via quadratic assignment problems (105–110); U. Rüde, Error estimates based on stable splittings (111–118); Marcus Sarkis, Multilevel methods for P_1 nonconforming finite elements and discontinuous coefficients in three dimensions (119–124); K. H. Tan and M. J. A. Borsboom, On generalized Schwarz coupling applied to advection-dominated problems (125–130); Olof B. Widlund, Exotic coarse spaces for Schwarz methods for lower order and spectral finite elements (131–136); S. F. Ashby, C. T. Kelley, P. E. Saylor and J. S. Scroggs, Preconditioning via asymptotically-defined domain decomposition (139–150); M. Azaïez and A. Quarteroni, A spectral Stokes solver in domain decomposition methods (151–156); Nikolai S. Bakhvalov [N. S. Bakhvalov] and Andrew V. Knyazev [Andreĭ Vladimirovich Knyazev], Preconditioned iterative methods in a subspace for linear algebraic equations with large jumps in the coefficients (157–162); Randolph E. Bank and Jinchao Xu, The hierarchical basis multigrid method and incomplete LU decomposition (163–173).

Tony F. Chan and Barry F. Smith, Domain decomposition and multigrid algorithms for elliptic problems on unstructured meshes (175–189); Clint N. Dawson and Mary F. Wheeler, Two-grid methods for mixed finite element approximations of nonlinear parabolic equations (191–203); Luc Giraud and Ray S. Tuminaro, Domain decomposition algorithms for PDE problems with large scale variations (205–210); Roland Glowinski, Tsorng-Whay Pan and Jacques Périaux, A one shot domain decomposition/fictitious domain method for the

Navier-Stokes equations (211–222); M. Griebel, Domain-oriented multilevel methods (223–229); Michael Holst and Faisal Saied, Multigrid and domain decomposition methods for electrostatics problems (231–238); Edgar Katzer, A parallel subspace decomposition method for hyperbolic equations (239–244); Seongjai Kim, Numerical treatments for the Helmholtz problem by domain decomposition techniques (245–250); Yu. A. Kuznetsov [Yuriĭ Alekseevich Kuznetsov], P. Neittaanmäki and P. Tarvainen, Schwarz methods for obstacle problems with convection-diffusion operators (251–256); C.-H. Lai [Choi Hong Lai], On domain decomposition and shooting methods for two-point boundary value problems (257–263).

W. Layton, J. Maubach and P. Rabier, Robust methods for highly nonsymmetric problems (265–270); Nancy J. Lybeck and Kenneth L. Bowers, Domain decomposition via the sinc-Galerkin method for second order differential equations (271–276); Petr Mejzlík, A bisection method to find all solutions of a system of nonlinear equations (277–282); Ilya D. Mishev, Preconditioning cell-centered finite difference equations on grids with local refinement (283–288); F. Nataf and F. Rogier, Outflow boundary conditions and domain decomposition method (289–293); J. Tinsley Oden, Abani Patra and Yusheng Feng, Domain decomposition for adaptive *hp* finite element methods (295–301); Michael Pernice, Domain decomposition preconditioners with Krylov subspace methods as subdomain solvers (303–309); L. Ridgway Scott, Elliptic preconditioners using fast summation techniques (311–323); Jian Ping Shao, The modified vertex space domain decomposition method for Neumann boundary value problems (325–336); Tsi Min Shih, Chin Bo Liem, Tao Lü and Ai Hui Zhou, A multi-color splitting method and convergence analysis for local grid refinement (337–342).

Olaf Steinbach, Boundary elements in domain decomposition methods (343–348); Victor H. Sun and Wei-Pai Tang, An overdetermined Schwarz alternating method (349–354); Xue Cheng Tai, Domain decomposition for linear and nonlinear elliptic problems via function or space decomposition (355–360); H. Wang, H. K. Dahle, R. E. Ewing, T. Lin and J. E. Våg, ELLAM-based domain decomposition and local refinement techniques for advection-diffusion equations with interfaces (361–366); Petter E. Bjørstad, W. M. Coughran, Jr. and Eric Grosse, Parallel domain decomposition applied to coupled transport equations (369–380); Nikos Chrisochoides, Geoffrey Fox and Joe Thompson, MENUS-PGG: a mapping environment for unstructured and structured numerical parallel grid generation (381–386); Patrick Ciarlet, Jr., A comparison of three iterative algorithms based on domain decomposition methods (387–393); E. De Sturler, IBLU preconditioners for massively parallel computers (395–400); Charbel Farhat and Po-Shu Chen, Tailoring domain decomposition methods for efficient parallel coarse grid solution and for systems with many right-hand sides (401–406); Hai Xiang Lin, Analysis and implementation of DD methods for parallel FE computations (407–412).

M. Reza Mehrabi and Robert A. Brown [Robert Arthur Brown], Finite-element/Newton method for solution of nonlinear problems in transport processes using domain decomposition and nested dissection on MIMD parallel computers (413–425); Mo Mu and John R. Rice, Modeling with collaborating PDE solvers: theory and practice (427–438); François-Xavier Roux and Damien Tromeur-Dervout, Parallelization of a multigrid solver via a domain decomposition method (439–444); Jonathan Richard Shewchuk and Omar Ghattas, A compiler for parallel finite element methods with domain-decomposed unstructured meshes (445–450); Renzo Arina and Claudio Canuto, A χ-formulation of the viscous-inviscid domain decomposition for the Euler/Navier-Stokes equations (453–458); Wei Cai, Domain decomposition and computation of two-dimensional detonation waves (459–464); X.-C. Cai, W. D. Gropp, D. E. Keyes and M. D. Tidriri, Parallel implicit methods for aerodynamics (465–470); Y. Cai and I. M. Navon, Parallel domain-decomposed preconditioners in finite element shallow water flow modeling (471–476); F. Camilli, M. Falcone, P. Lanucara and A. Seghini, A domain decomposition method for Bellman equations (477–483); J. G. Chefter, C. K. Chu and D. E. Keyes, Domain decomposition for the shallow water equations (485–490).

R. K. Coomer and I. G. Graham, Domain decomposition methods for device modelling (491–496); George C. Hsiao, Michael D. Marcozzi and Shangyou Zhang, An efficient computational method for the flow past an airfoil (497–502); P. G. Jacobs, V. A. Mousseau, P. R. McHugh and D. A. Knoll, Newton-Krylov-Schwarz techniques applied to the two-dimensional incompressible Navier-Stokes and energy equations (503–507); H. C. Ku, H. E. Gilreath, R. Raul and J. C. Sommerer, Direct numerical simulation of jet flow via a multiblock technique (509–514); Patrick Le Tallec, Jan Mandel and Marina Vidrascu, Balancing domain decomposition for plates (515–524); S. Micheletti, A. Quarteroni and R. Sacco, Nonlinear block iterative solution of semiconductor device equations by a domain decomposition method (525–531); I. Raspo, J. Ouazzani and R. Peyret, A direct Chebyshev multidomain method for flow computation with application to rotating systems (533–538); L. Vozovoi, M. Israeli and A. Averbuch, Multi-domain Fourier algorithms for parallel solution of the Navier-Stokes equations (539–546).

{The papers of mathematical interest that appear to be in final form are being reviewed individually.}

MR1282742 (95c:76083) 76M25 76S05 86A05

Ewing, Richard E. (1-TXAM-IS);
Wang, Hong [**Wang, Hong 9**] (1-TXAM-IS)

Eulerian-Lagrangian localized adjoint methods for variable-coefficient advective-diffusive-reactive equations in groundwater contaminant transport. (English. English summary)

Advances in optimization and numerical analysis (*Oaxaca*, 1992), 185–205, *Math. Appl.*, 275, *Kluwer Acad. Publ.*, *Dordrecht*, 1994.

Summary: "In this paper, we develop Eulerian-Lagrangian localized adjoint methods (ELLAMs) to solve variable-coefficient advective-diffusive-reactive transport equations governing contaminant transport in groundwater flowing through a porous medium. The derived numerical schemes can treat various combinations of boundary conditions and conserve mass. Moreover, our numerical schemes provide an alternative approach to reducing the dependence of the numerical solutions on the accurate tracking of the characteristics, which is essential in many Eulerian-Lagrangian methods (ELMs). Numerical results are presented to demonstrate the strength of our schemes."

{For the entire collection see MR1282730 (95a:00010)}

MR1259218 (94j:65119) 65M60

Herrera, G. (1-VT-EM); **Herrera, I.** (MEX-NAM-GP)

Eulerian-Lagrangian method of cells based on localized adjoint method. (English. English summary)

Numer. Methods Partial Differential Equations **10** (1994), *no.* 2, 205–223.

Summary: "The localized adjoint method, when applied using an Eulerian-Lagrangian frame, has been quite successful in treating advection-dominated transport. The resulting methodology is known as ELLAM. In previous work, bilinear functions were used as test functions. In this paper, locally constant functions are used instead, leading to procedures which are appealing because, in addition to other advantages of ELLAM methods, they ensure local mass conservation, are easy to apply and can be combined without difficulty with existing solute-transport codes which are based on finite volumes. In addition, the procedures for deriving the algorithms presented here

are used as an illustration of a general methodology for numerically treating partial differential equations which is advocated by the authors. This methodology consists in identifying the information about the sought solution which is contained in the approximate one and then using this insight to choose the interpolation procedure to be applied."

MR1223256 (94g:65098) 65M50 65N50 76M25
Herrera, Ismael (MEX-NAM-GP)**;**
Ewing, Richard E. (1-TXAM-SC)**;**
Celia, Michael A. (1-PRIN-WT)**; Russell, Thomas F.** (1-COD)
Eulerian-Lagrangian localized adjoint method: the theoretical framework. (English. English summary)
Numer. Methods Partial Differential Equations **9** (1993), *no.* 4, 431–457.
Summary: "This is the second of a sequence of papers devoted to applying the localized adjoint method (LAM), in space-time, to problems of advective-diffusive transport. We refer to the resulting methodology as the Eulerian-Lagrangian localized adjoint method (ELLAM). The ELLAM approach yields a general formulation that subsumes many specific methods based on combined Lagrangian and Eulerian approaches, so-called characteristic methods. In the first paper of this series ["An Eulerian-Lagrangian localized adjoint method for the advection-diffusion equation", Adv. Water Res. 13 (1990); per bibl.] the emphasis was placed on the numerical implementation, and a careful treatment of implementation of boundary conditions was presented for one-dimensional problems. The final ELLAM approximation was shown to possess the conservation of mass property, unlike typical characteristic methods. The emphasis of the present paper is on the theoretical aspects of the method. The theory, based on Herrera's algebraic theory of boundary value problems [Boundary methods, Pitman, Boston, MA, 1984; MR0766561 (86m:35006)], is presented for advection-diffusion equations in both one-dimensional and multidimensional systems. This provides a generalized ELLAM formulation. The generality of the method is also demonstrated by a treatment of systems of equations as well as a derivation of mixed methods."

MR1179165 (93e:76058) 76M25 76M30

Herrera, Ismael (MEX-NAM-GP)

Localized adjoint methods: a new discretization methodology. (English. English summary)

Computational methods in geosciences, 66–77, *SIAM*, *Philadelphia*, *PA*, 1992.

Summary: "The localized adjoint method (LAM) is a new and promising methodology of wide applicability, based on our algebraic theory of boundary value problems. In this paper, the general theory is briefly explained and then its application is illustrated with transport diffusion problems for which the Eulerian-Lagrangian localized adjoint method (ELLAM) has been formulated by the LAM group (M. A. Celia, R. E. Ewing and T. F. Russell, in addition to the author). The ELLAM development unifies characteristic methods and treats boundary conditions systematically, yielding conservative schemes."

For the entire collection see MR1179163 (93d:86001).

{For the entire collection see MR1179163 (93d:86001)}

MR1132500 (93b:76082) 76S05 76M25 76T05

Ewing, Richard E. (1-WY)

Operator splitting and Eulerian-Lagrangian localized adjoint methods for multiphase flow.

The mathematics of finite elements and applications, *VII* (*Uxbridge*, 1990), 215–232, *Academic Press*, *London*, 1991.

The paper under review is based on earlier work of the author and should be considered as a summary of this work.

In Section 2 the analogy between miscible and two-phase immiscible flow equations is established. A similar concept was used by K. Aziz and A. Settari [*Petroleum reservoir simulation*, Appl. Sci. Publ., London, 1979] but is not popular in petroleum engineering applications. In the next sections two methods of weak solution of the two-phase flow equations (called operator splitting and ELLAM) are presented and discussed. The above problem is very important in practical applications, e.g., in oil reservoir simulation. The modified method presented here may be very useful, especially for the long-term simulation where the Courant number is greater than 1. The paper is difficult to read because of many references, e.g., the parameters $S, \overline{S}, \tilde{S}$ in (2.7), (3.5) are undefined, the boundary conditions for equation (2.5) in the case of two-phase immiscible flow are not evident, the $C^n(c)$ in (3.3) is not clear (the last is probably a mistake). There are no numerical experiments in the paper. In the one-dimensional case of the two-phase immiscible flow with $D \to 0$, where D is the diffusion term, the numerical results could be compared with the analytical/graphical Welge solution (commonly used in practical applications) and/or with the numerical finite difference solution using, e.g., the Lax method.

{For the entire collection see MR1132483 (92d:65007)}

Jerzy Stopa (Kraków)