M331 Introduction to Mathematical Modeling
Final Examination

Closed book, closed neighbor. December 12, 2000

1. Consider a 20-lb pink flamingo that stands 3ft in height and has legs that are 2ft in length. Model the height and leg length of a 100-lb flamingo. What assumptions are necessary? Are they reasonable assumptions?

2. A certain drug is effective in treating a disease if the concentration remains above 250 mg/L. The initial concentration is 640 mg/L. It is known from laboratory experiments that the drug decays at the rate of 20% of the amount present at each hour.

   - Formulate a model representing the concentration at each hour.
   - Build a table of values and determine when the concentration reaches 250 mg/L.

3. Derive the equations that minimize the sum of the squared deviations between a set of data points and the quadratic model

$$y = c_1 x^2 + c_2 x + c_3$$

4. Consider the gasoline inventory problem. In many situations the time T between deliveries and the order quantity Q is not fixed. Instead, an order is placed for a specific amount of gasoline. Depending on how many orders are placed in a given time interval, the time to fill an order varies. You have no reason to believe that the performance of the delivery operation will change. Therefore, you have examined records for the last 100 deliveries and found the following lag times or extra days required to fill your order:

Lag time (in days)	Number of Occurrences
2	10
3	25
4	30
5	20
6	13
7	2

Provide pseudocode for a Monte Carlo simulation for the lag time submodel. You do not actually have to generate actual numbers from this simulation.

5. A local company restores cars and trucks for resale. Each vehicle must be processed in the refinishing-paint shop and the machine-body shop. Each car contributes on average \$3,000 to profit and each truck contributes \$2,000 to profit. The refinishing-paint shop has 2400 work hours and the machine body shop has 2500 work-hours available. A car requires 50 work-hours in the machine-body shop and 40 work-hours in the refinishing paint shop. A truck requires 50 work-hours in the machine-body shop and 60 work-hours in the refinishing paint shop. Use graphical linear programming to determine a daily production schedule that will maximize the company's profits.

6. One general rule for roasting a turkey is the following: Set the oven to 400 degrees F and allow 20 minutes per pound for cooking. How good a rule is this? Use dimensional analysis and the concept of geometric similarity to evaluate this rule. You may assume that the relevant parameters include

   - l, the characteristic length scale of the turkey
   - ΔT_m, the temperature difference between the raw meat and the oven
   - ΔT_c, the temperature difference between the cooked meat and the oven
   - k, the thermal conductivity of the turkey
   - t, the cooking time

   The required dimensions of each variable are:

Variable	ΔT_m	ΔT_c	k	l	t
Dimension	$ML^{-1}T^{-2}$	$ML^{-1}T^{-2}$	L^2T^{-1}	L	T