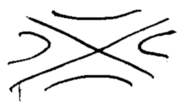
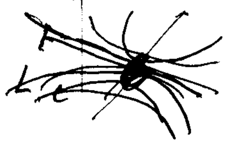
8. **(40 points)**

Classify the type of phase plane portrait of $\mathbf{x}' = \mathbf{A}\mathbf{x}$ for the following matrices $\mathbf{A}$. Also state whether the origin is stable or unstable. [**XC 5 pts.: Sketch the phase portrait**]

a) 

$$\mathbf{x}' = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \mathbf{x},$$

$T = 0$, $D = -2$, $T^2 - 4D = 8 > 0$
Saddle, unstable.

b) 

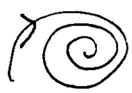
$$\mathbf{x}' = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \mathbf{x},$$

$T = 3$, $D = 1$, $T^2 - 4D = 5 > 0 \Rightarrow$ nodal source, unstable

c) 

$$\mathbf{x}' = \begin{bmatrix} 2 & 2 \\ -4 & -2 \end{bmatrix} \mathbf{x},$$

$T = 0$, $D = 4$, $T^2 - 4D < 0 \Rightarrow$ center
neutrally stable

d) 

$$\mathbf{x}' = \begin{bmatrix} 1 & 4 \\ -2 & -3 \end{bmatrix} \mathbf{x}.$$

$T = -2$, $D = 5$, $T^2 - 4D = -16 < 0$
$\Rightarrow$ spiral sink, stable.

XC (10 points) Consider the equation

$$t^2 y'' - 2y = 2t^3$$

Show that $\{t^2, t^{-1}\}$ is a fundamental set of solutions for the homogeneous equation. Find the general solution of the nonhomogeneous equation.

a) show t^2, t^{-1} are solutions of the homogeneous eq.

b) show $W_{\{t^2, t^{-1}\}}(t) \neq 0$,

c) find a particular solution of the nonhomogeneous equation, then

$y_G = y_H + y_p$, $y_H = c_1 t^2 + c_2 t^{-1}$

$y_p(t) = ?$;)