36) Show that $G = \langle x, y \mid x^2, y^2 \rangle$ is infinite.
Hint: Find a group (e.g. in $\mathrm{GL}(2, \mathbb{Q})$) that must be a quotient of G, but contains elements of infinite order.

37) a) Determine a presentation for S_3 on the generators $a = (1, 2, 3)$ and $b = (2, 3)$.
b) Determine a presentation for S_3 on the generators $a = (1, 2, 3)$, $b = (2, 3)$ and $c = (1, 2)$.
c) Determine a presentation for S_3 on the generators $c = (1, 2)$ and $b = (2, 3)$.
d) Using that $S_4 = C_2^2 \rtimes S_3$, determine a presentation for S_4.

38) Select generators for S_5 and find sufficiently many identities amongst the generators to create a presentation for S_5. You may use GAP to verify that the group given by your presentation has order 120.

39) Show that every finitely generated group G has only finitely many subgroups of index n for any given n.
Hint: Every subgroup of index n gives rise to a homomorphism $G \to S_n$, described completely by the generator images. Show that there are just finitely many possibilities for generator images.
Can you find a counterexample of an infinitely generated G?

40) (J. P. Serre) Let F be the free group on 26 generators $a, b, \ldots$. We create relations $l = r$ if l and r are English words that sound the same but are spelled differently (assume "news reader" English) (for example $see = sea$). What can you tell about the structure of the finitely presented group given by these relations?

41) Let G be a subdirect product of the groups $A := G^\alpha$ with $B := G^\beta$ and let $S, T \leq G$. Describe reductions for the following calculations to calculations in A and B:

a) $S \cap T$

b) $N_T(S)$

c) Test $S \leq T$.