36) Show that $G = \langle x, y \mid x^2, y^2 \rangle$ is infinite. 

**Hint:** Find a group (e.g. in $\text{GL}(2, \mathbb{Q})$) that must be a quotient of $G$, but contains elements of infinite order.

37) a) Determine a presentation for $S_3$ on the generators $a = (1, 2, 3)$ and $b = (2, 3)$.
   b) Determine a presentation for $S_3$ on the generators $a = (1, 2, 3)$, $b = (2, 3)$ and $c = (1, 2)$.
   c) Determine a presentation for $S_3$ on the generators $c = (1, 2)$ and $b = (2, 3)$.
   d) Using that $S_4 = C_2 \times S_3$, determine a presentation for $S_4$.

38) Select generators for $S_5$ and find sufficiently many identities amongst the generators to create a presentation for $S_5$. You may use GAP to verify that the group given by your presentation has order 120.

39) Show that every finitely generated group $G$ has only finitely many subgroups of index $n$ for any given $n$.

**Hint:** Every subgroup of index $n$ gives rise to a homomorphism $G \rightarrow S_n$, described completely by the generator images. Show that there are just finitely many possibilities for generator images. Can you find a counterexample of an infinitely generated $G$?

40) (J. P. Serre) Let $F$ be the free group on 26 generators $a, b, \ldots$. We create relations $l = r$ if $l$ and $r$ are English words that sound the same but are spelled differently (assume “news reader” English) (for example see = sea). What can you tell about the structure of the finitely presented group given by these relations?

41) Let $G$ be a subdirect product of the groups $A := G^\alpha$ with $B := G^\beta$ and let $S, T \leq G$. Describe reductions for the following calculations to calculations in $A$ and $B$:

   a) $S \cap T$
   b) $N_T(S)$
   c) Test $S \leq T$. 