42) If you define a finitely presented group in GAP, you can enforce reduction of elements to normal form by calling `SetReducedMultiplication(G);` Why do you think this is not turned on by default?

43) Let M be the monoid with the presentation $\langle x,y \mid x^3=1, y^3=1, (xy)^3=1\rangle$.
a) Determine (by hand) a confluent rewriting system for M with respect to length+lexicographic ordering.
b) Using the rules determined in a), construct an infinite set of words in normal form, thus proving that M is infinite.

44) A *Dynkin diagram* is a graph whose edges (i,j) are labelled with numbers $e_{i,j}\in\{3,4,\ldots\}$. We set $e_{i,j}=2$ if i and j are not connected. We now associate to such a diagram on the vertices $1,\ldots,n$ a finitely presented group (called the "Weyl group" or "Coxeter group" for this diagram) on n generators $a_1,\ldots,a_n$ with the relations: $a_i^2=1$ and for $i<j$:

$$\underbrace{a_j a_i a_j a_i\cdots}_{e_{i,j}\text{ letters}} = \underbrace{a_i a_j a_i a_j\cdots}_{e_{i,j}\text{ letters}}$$

Show that this presentation is a confluent rewriting system.

45) The Dynkin diagram called A_n has the form $0-0-0-\cdots-0$ with n vertices [1]. So for example for $n=3$ we get the presentation

$$\langle a_1,a_2,a_3 \mid a_1^2=a_2^2=a_3^3=1, a_2a_1a_2=a_1a_2a_1, a_3a_1=a_1a_3, a_3a_2a_3=a_2a_3a_2\rangle$$

Show that the Weyl group of type A_n is isomorphic to the symmetric group S_{n+1}.

46*) a) Why is a pure lexicographic ordering not a reduction ordering? Give a (counter)example.
b) Prove that the wreath product ordering of reduction orderings is again a reduction ordering.

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[1] A politician (in the original version the head of the Soviet Union's central committee) is giving a speech to open the Olympic Games: "O! O! O! O! O!".
An aide comes over and whispers: "Sir, those are the Olympic rings, your speech is below!"