39) (corrected) a) Let S be a positive definite symmetric bilinear form on R^n, given by (as in problem 38) a matrix A (this matrix is called the Gram matrix for S with respect to the standard basis). . Let $C \in \mathbb{R}$. Show that there are bounds $c_i = c_i(C)$ (depending on A and C) so that if $\underline{\mathbf{x}} = (x_1, \ldots, x_n) \in \mathbb{Z}^n$ with $S(\underline{\mathbf{x}}, \underline{\mathbf{x}}) = \underline{\mathbf{x}}^T A \underline{\mathbf{x}} \leq C$, then $|x_i| \leq c_i$.
(Hint: Consider the problem first for an orthogonal basis.) b) Let

$$A = \begin{pmatrix} 89 & -433 & -75 \\ -433 & 2109 & 371 \\ -75 & 371 & 79 \end{pmatrix}.$$

Determine all vectors in $\mathbb{Z}^n$ with $S(\underline{\mathbf{x}}, \underline{\mathbf{x}}) \leq 4$.
Note The `.remainder` component of the result of `LLLReducedGramMat` (see the online help or the manual) contains the Gram matrix for S with respect to a basis reduced with respect to S. If you work in this new basis, coefficients stay smaller.

41) Determine all integral solutions $\underline{\mathbf{x}} \in \mathbb{Z}^3$ of the system

$$\begin{pmatrix} 366 & 133 & -1564 \\ -290 & -105 & 1240 \\ 5 & 0 & -25 \end{pmatrix} \cdot \underline{\mathbf{x}} = \begin{pmatrix} -899 \\ 715 \\ -25 \end{pmatrix}$$

(Hint: Use lattice reduction first to keep coefficients small, then use a Smith Normal Form)

42) Show that the grlex (graded lexicographic) ordering is a monomial ordering.

43) Rewrite the following polynomial, ordering its terms according to the lex, grlex and grevlex ordering and give $lm(f)$, $lc(f)$ and multideg(f) in each case.

$$f(x,y,z) = 2x^2y^8 - 3x^5yz^4 + xyz^3 - xy^4.$$

4) Let $B = (x^2y - z, xy - 1)$ and $f = x^3 - x^2y - x^2z + x$. a) Compute the remainder when dividing f by the elements of B for the lex ordering.
b) Repeat part a) with the order of the pair B reversed.

Problems marked with *are bonus problems for extra credit.

From April 7 on, we will also meet Mondays, at 9am in Weber 14 to make up for lost lectures.