26) Let $A = \mathbb{Z}$ and $M$ a finitely generated abelian group (i.e. a $\mathbb{Z}$-module). Identify the associated primes of $M$ in terms of the structure theory for finitely generated abelian groups.

27) Let $J \triangleleft A$ and $J \subset P, Q \triangleleft A$. Show that $Q$ is a $P$-primary ideal of $A$ if and only if $Q/J$ is a $P/J$-primary ideal of $R/J$.

28) (GAP) Let $A = \mathbb{Q}[x, y, z]$. Show that

$$J = \left((y^2 - xz)(z^2 - x^2y), (y^2 - xz)z\right) = (y^2 - xz) \cap (x^2, z) \cap (y, z^2)$$

is a minimal primary decomposition of $J$. Determine the irreducible and the embedded prime ideals associated to $J$.

29) Let $A = \mathbb{Q}[x, y]$ and $J = (x^3, xy) \triangleleft A$.
   a) Show that for every $n \in \mathbb{N}$ the ideal $(x^3, xy, y^n) \triangleleft A$ is primary.
   b) Show that $J = (X) \cap (x^3, y)$ is a minimal primary decomposition of $J$.
   c) Construct infinitely many minimal primary decompositions of $J$.

30) Let $f: A \to B$ be a surjective ring homomorphism. Let $J \triangleleft B$ and $I = f^{-1}(J) \triangleleft A$.
   a) Show that $I$ is a primary ideal of $A$ if and only if $I^e$ is a primary ideal of $B$.
   b) Show that (if $I$ is primary) that $r(I^e) = r(I)^e$ and $r(I) = (r(I^e))^e$.

31) Let $M \triangleleft k[x_1, \ldots, x_n]$ be a monomial ideal. When is $M$ primary?