Mathematics 666 **Homework (due Oct. 29)** A. Hulpke

34) We want to show that induction is transitive, i.e. if $S \le T \le G$ and φ is a representation of S, then $\varphi\uparrow_S^G$ is similar to $(\varphi\uparrow_S^T)\uparrow_T^G$.
We will do so on the level of characters, i.e. show that if χ is the character of φ, then

$$\chi\uparrow_S^G = (\chi\uparrow_S^T)\uparrow_T^G$$

35) Let G be a finite group and $N \lhd G$, $\chi \in \mathrm{Irr}(G)$ and 1_N the trivial character of N. Show:
If $(1_N, \chi\downarrow_N) \neq 0$, then $N \le \ker(\chi)$.

36) a) Let $U \le G$ and $\chi \in \mathrm{Irr}(U)$. Is χ always a constituent of $(\chi\uparrow^G)\downarrow_U$, i.e. of the restriction of $\chi\uparrow^G$ to U? (Proof or counterexample.)
b) Does the result in a) depend on χ being irreducible?

37) (followup on problem 21) Let $G = \mathrm{SL}_3(2)$. (This is up to isomorphism the only simple group of order 168.) We want to determine the character table of G. The following steps indicate one possible way, but you may try other methods:
You have determined class sizes and centralizer orders in problem 21, here is the corresponding table header with centralizer orders, classes and power maps.

```
     168  8  3  4  7  7

        1a 2a 3a 4a 7a 7b
     2P 1a 1a 3a 2a 7a 7b
     3P 1a 2a 1a 4a 7b 7a
     7P 1a 2a 3a 4a 1a 1a
```

a) Determine the permutation character for the permutation action of G on the seven nonzero vectors in ${\mathbb{F}_2}^3$. Show that its nontrivial part is irreducible.
b) Let S be the stabilizer of the first basis vector. Show that S must have order 24. Let $T \le S$ the subgroup of block diagonal matrices with blocks of size 1 and 2. Show that $T \cong S_3 \cong \mathrm{SL}_2(2)$ and that the action of S on the cosets of T gives an isomorphism $S \cong S_4$.
c) Calculate the induced characters $\chi\uparrow^G$ for all $\chi \in \mathrm{Irr}(S_4)$. Reduce with the irreducible characters already found. This should give you two new irreducible characters of degrees 7 and 8.