38) Let G be a finite group and θ a permutation character for a transitive action of G of degree > 1. Show that there exists $g \in G$ such that $\theta(g) = 0$.

39) (GAP) Let $G = M_{11}$. From the example in class we know that G has a possible permutation character of degree 12, that is action on the cosets of a subgroup of order 660. Determine the character table of such a subgroup. What can you tell about this subgroup's structure?

40) Let $S \leq G$, $g \in G$, and $T = S^g \leq G$. Let χ be a character of S.
a) We define the function ψ on T by

$$\psi(t) := \chi(g \cdot t \cdot g^{-1}) = \chi(t^{(g^{-1})}).$$

Show that ψ is a character of T.
b) Show that $\chi \uparrow_S^G = \psi \uparrow_T^G$.

41) Let n be a positive integers, $k \leq \frac{n}{2}$, and Ω_k the set of all k-sets with elements from $1 \ldots n$, that is $|\Omega_k| = \binom{n}{k}$. Let π_k be the character of the permutation representation of S_n on Ω_k.
a) Show that for $l \leq k$ we have that

$$(\pi_k, \pi_l) = l + 1.$$

(Hint: Consider the action of S_n on $\Omega_k \times \Omega_l$.)
b) Show that $\chi_k := \pi_k - \pi_{k-1}$ is an irreducible character and that for $l \neq k$ we have that $\chi_k \neq \chi_l$.
c) For $n = 5$, calculate the values of χ_3 and χ_4 on the classes of the elements $(1, 2)$ and $(1, 2, 3)$.